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Monetary policy in an era of global supply chains☆

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ABSTRACT

We study the implications of global supply chains for the design of monetary policy, using a small-open economy New Keynesian model with multiple stages of production. Within the family of simple monetary policy rules with commitment, a rule that targets separate producer price inflation at different production stages, in addition to output gap and real exchange rate, is found to deliver a higher welfare level than alternative policy rules. As an economy becomes more open, measured by export share, the optimal weight on the upstream inflation rises relative to that on the final stage inflation. If we have to choose among aggregate price indicators, targeting PPI inflation yields a smaller welfare loss than targeting CPI inflation alone. As the production chain becomes longer, the optimal weight on PPI inflation in the policy rule that targets both PPI and CPI inflation will also rise. A trade cost shock such as a rise in the import tariff can alter the optimal weights on different inflation variables.

1. Introduction

We live in a world of supply chains. From the data of World Input-output Tables, in 2000, the world gross output was 1.97 times that of the world value added, suggesting a large role of intermediate inputs in production and supply chains in the modern economy. Many supply chains are global. Trade in intermediate goods has been growing faster than trade in final goods. The importance of supply chains has also grown over time; the ratio of gross output to value added has increased to 2.18 by the end of 2014. In this paper, we study the implications of global supply chains for the design of optimal monetary policy.

There is an active research on outsourcing and offshoring in the field of international trade, where firms purchase intermediate inputs from other firms, sometimes foreign firms, for further processing.[1] Global supply chains are rising in importance as an increased fraction of output is produced as intermediate inputs rather than final consumption. As important, it is accompanied by an increase in the number of production stages in many sectors (e.g., Wang et al., 2017). The role of globalization in national inflation behavior has also received increased attention.[2]

A voluminous but separate literature in monetary economics studies optimal monetary policy. Woodford (2010) provides an excellent survey of the subject in an closed-economy setting, whereas Corsetti et al. (2010) supply an excellent survey of issues in the new open-economy macroeconomics. While central banks typically target only

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1 See Feenstra (1999), Hummels et al. (2001), Yi (2003), Koopman et al. (2014), Antràs (2016), and Johnson and Noguera (2016), among others.
2 Recent examples include Auer et al. (2017a), Auer et al. (2017b), Forbes (2018), and Wei and Xie (2019).

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CPI inflation, the literature has studied whether CPI or PPI is more appropriate for monetary policy goal (e.g., De Paoli, 2009; De Gregorio, 2012). Two pioneering papers are especially worth noting. In an open-economy model featuring a single stage of production (i.e., no supply chains), Galí and Monacelli (2005) suggest that PPI is a better target, where PPI in their context is the price index for domestically produced final products. In a closed-economy featuring two stages of production (i.e., there are simple national supply chains but not global supply chains), Huang and Liu (2005) demonstrate that the optimal simple rule should include PPI inflation as well as CPI, where PPI is the producer prices of domestically produced intermediate products.[3] The intuition is that, in a New Keynesian model, a PPI inflation causes distortions in the allocation of productive resources, including among domestic producers of intermediate goods. Since all firms are owned by the households, the distortions associated with a PPI inflation reduce household welfare too.

Interactions between multi-stage production and economic openness and their implications for the design of monetary policy have not been much explored. For example, when an economy becomes more open, should the optimal weights on the upstream sector inflation rise or fall relative to those on the final stage inflation? Should trade frictions such as a rise in the tariff rate affect the design of monetary policy?

We build a New Keynesian model that features simultaneously multi-stage production and openness. A noteworthy feature of the equilibrium is that there are separate Phillips curve relationships for each production stage that link the producer price inflation of a given stage to both the expected next-period inflation and log-deviation of that stage's real marginal cost from the steady state. The real marginal cost term for each production stage, in turn, is a function of change in the real exchange rate (due to the openness of the economy) and a relative price gap between the production stages (due to multiple stages of production).

Following Rotemberg and Woodford (1999), Galí and Monacelli (2005), and Huang and Liu (2005), we assume that the central bank maximizes the welfare of the household which is approximated by a second order expansion of the utility function. By making use of equilibrium conditions, we can see that the welfare loss function contains not only output gap and change in the real exchange rate, but also separate producer price inflation in each production stage, separate terms for employment fluctuations in each production stage, and the relative price gap between the production stages. Parameters describing the openness of the economy (shares of each sector's output that are sold abroad and shares of inputs imported from abroad) appear in the welfare loss function as well.

Quantitatively, we estimate the nonlinear model up to second-order expansion (of both the constraints and the welfare function), and consider a family of simple monetary rules, including (a) the classic Taylor (1993) rule that features output gap, CPI inflation, and change in the real exchange rate, (b) the Galí and Monacelli (2005) rule in which PPI inflation takes the place of CPI inflation in the Taylor rule, (c) a rule that includes separate producer price inflation in each stage of production as well as output gap and change in the real exchange rate, and (d) some variations of the previous rules that omit either output gap, change in the real exchange rate, or both. For each rule, we compute optimal weights on each term in the monetary policy rule.

Within the family of simple monetary policy rules, a rule that targets separate producer price inflation at each stage of production (as well as output gap and change in the real exchange rate) delivers a higher welfare level than alternative policy rules. As an economy becomes more open, measured by the share of exports in sales, the optimal weight on the upstream sector inflation rises relative to that on the final stage inflation.

Greater trade frictions reduce the openness of an economy. This, in the model, would dampen the optimal weight on the upstream sector inflation. However, we document a direct welfare loss associated with greater trade frictions even if the monetary policy rule adjusts optimally. In other words, the central bank cannot completely offset the negative effects of greater trade frictions. Naturally, the welfare loss would be even greater without the re-optimization by the central bank.

In general, because the optimal weights on the inflation at different production stages are not proportional to the sales weights, the PPI inflation would not be sufficient to replace these production-stage-specific inflation. At the same time, if we restrict ourselves to only consider aggregate inflation measures (PPI and CPI), targeting PPI inflation yields a smaller welfare loss than targeting CPI inflation alone (in addition to output gap and change in the real exchange rate). That is because PPI inflation puts at least some weight on the upstream sector inflation whereas CPI inflation puts none.

We also consider a general version of the model that can feature an arbitrary number of production stages (but in a closed economy). In this case, as the number of production stages increases, the optimal weights on the upstream sector inflation as a whole relative to the final stage of production, or the optimal weight on the PPI inflation (if we only consider aggregate price index), will increase as well. This discussion is collected in an appendix.

Is it possible for countries to obtain separate producer price index for upstream and downstream sectors? In turns out that the United States, Japan, Australia, Korea and Canada already collect and report such data. For instance, the US Bureau of Labor Statistics (BLS) has considered a four-stage production process and accordingly constructed a stage-of-processing price indices in the PPI Final Demand-Intermediate Demand indice.[4] Fig. 1 presents separate inflation paths for producer price indices at different production stages as well as core CPI for the United States (Panel A) and Australia (Panel B). It can be seen clearly that the producer price inflation in the upstream sectors and the final stage do not move together**.** This means that the monetary policy implied by a rule that includes separate producer price inflation would look different from the classic Taylor rule.

This paper builds on the literature on monetary policy by introducing global supply chains. Corsetti et al. (2010) provide a comprehensive survey on early literature. Galí and Monacelli (2005) build a small-open economy New Keynesian model that features a single stage of production, and compare three alternative simple policy rules: CPI-based Taylor rule, PPI-based rule, and an exchange rate peg. De Paoli (2009) demonstrates in a model with more general parameterization but also a single stage of production and focuses on terms of trade externality in driving the optimal monetary policy. Shi and Xu (2007) build a two-country New Keynesian model with trade in vertical production, focusing on transborder spillover effect of productivity shock and the discussion of optimal money supply policy. To explain international business cycles, Huang and Liu (2007) build a two-stage production model with staggered prices. Aoki (2001), among early works, studies the optimal sectoral weights in the monetary policy rule when there are two horizontal sectors. Lombardo and Ravenna (2014) study optimal monetary policy in a two (horizontal) sectors under one stage of production with imported inputs for the tradable sector. Matsumura (2019) also studies monetary policy in a small-open economy with multiple sectors but still with only one stage of production.

Our point of departure from this set of papers is a simultaneous introduction of multi-stage production and economic openness. We pay special attention to an interaction between openness and multi-stage production and its implication for the monetary policy rule. In a closed economy setting with two stages of production, Huang and Liu (2005) show that a monetary policy rule that includes PPI inflation is preferred to targeting CPI inflation alone. This is true in our generalized model as

[3] Strum (2009) expands on the model with two-stage production developed in Huang and Liu (2005) and revisits the classic question of optimal commitment versus discretionary monetary policies.

[4] Details about PPI Final Demand-Intermediate Demand indices can be found at https://www.bls.gov/ppi/fdidsummary.htm.

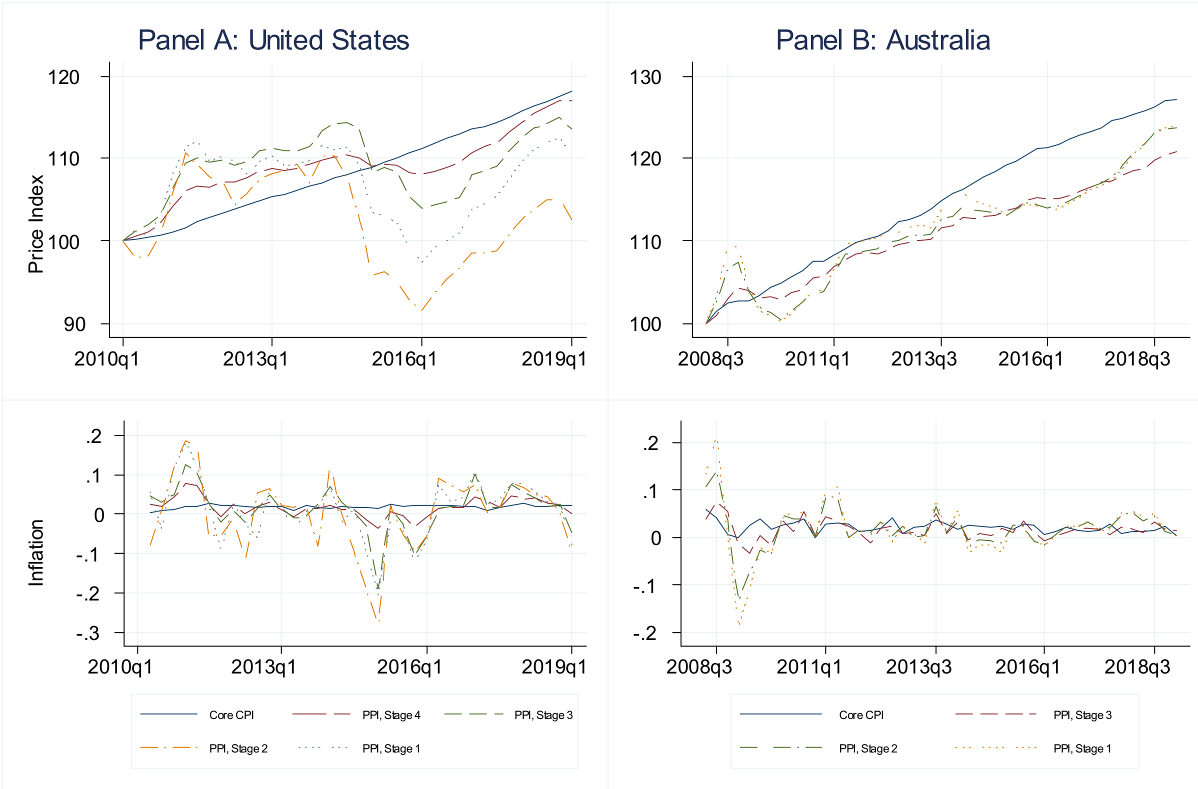

Fig. 1. Stage-of-processing producer price index and core CPI, US and Australia.

well. In addition, we show that the degree of openness systematically affects the optimal weights on the early stage producer price inflation. We also use the model to discuss how the monetary policy rule may be affected by a sharp increase in the costs of international trade such as in a trade war. In a long appendix, we also feature an arbitrary number of production stages (in a closed economy) and investigate effects of a lengthening of the supply chains on the design of the monetary policy.

Gong et al. (2016) study optimal simple monetary policy rules in a two-country New Keynesian model with two stages of production. However, since labor is assumed to be only used in the first stage of production in their paper, there is no misallocation of labor across production stages. In other words, there is no resource misallocation across production stages due to stage-specific producer price inflation. In comparison, we do allow for potential misallocation across production stages. This generalization qualitatively changes the results of the analysis.

There are other issues discussed in the literature that we do not discuss here. For example, commitment versus discretion in the monetary policy (e.g., Strum, 2009) and the role of investment goods' prices (e.g., Basu and De Leo, 2019) can in principle be incorporated in our framework as extensions.

This paper is also related to a literature on the effects of globalization on national inflation. Auer et al. (2017a), Auer et al. (2017b), and Forbes (2018) study how national inflation dynamics are altered by intercountry connections through supply chains, and how the trade-offs in inflation targeting policies may be changed for central banks. Wei and Xie (2019) demonstrate how an increase in the number of production stages can lead to a weakening in the correlation between PPI and CPI inflation. However, that literature does not explore how an interaction between multi-stage production and globalization affects the design of the monetary policy.

The rest of the paper proceeds as follows: Section 2 introduces the basic model with global supply chains; Section 3 characterizes the steady-state, flexible-price, and sticky-price equilibrium in the special case of two stages of production, derives an expression for the welfare loss function, and discusses the comparative statics of changes in import tariff; Section 4 compares several monetary policy rules via calibrations; finally, Section 5 concludes the paper. An appendix discusses a more general case with an arbitrary number of production stages.

2. The model setting

Consider a small-open economy New Keynesian model with an infinitely lived representative household. The household maximizes its utility through consumption and leisure. The household owns all domestic firms and receives dividends from them.

The production of a final good requires N-stages of production, which constitutes a vertical production chain. In each stage of production, a large number of domestic firms produce a unit continuum of differentiated outputs, i.e., $u \in [0, 1]$. In the first stage of production, firms use domestic labor as the only input. In each of the subsequent stages of production, intermediary inputs purchased from the previous stage (from both domestic and foreign sources) together with labor are used together for production. All production features constant returns to scale.

The firms and households take international prices of inputs and foreign demand of outputs as given (the small open economy assumption). While the firms are price-takers in the factor markets, they are assumed to be monopolistic competitive in their outputs and set the output prices in their own currency (the producer currency pricing assumption, or the PCP). In future work, alternative assumptions such as pricing to the market and the dollar currency pricing can be explored.[5]

[5] Engel (2011) offers a detailed discussion on the implications of the local currency pricing. Mukhin (2018), Egorov and Mukhin (2019), and Gopinath et al. (2019) make a case for the dominance of US dollar pricing.

2.1. Household

The representative household has the following utility function and budget constraint:

$$E\Sigma_{t=0}^{\infty}\beta^t[U(C_t)-V(L_t)]$$
$$s.t.\ \ P_tC_t+E_t\{D_{t,t+1}B_{t+1}\}\leq W_tL_t+\Pi_t+B_t$$

where the variable C_t is the final consumption good, L_t is the supply of labor, $D_{t,t+1}$ is the price of a one-period nominal bond paying off in domestic currency, W_t is the nominal wage, Π_t includes both firm profits and a lump-sum transfer of any government tax revenue, and B_t denotes the holding of a riskless one-period bond.

The consumption good is a composite of both domestically produced and imported final goods, i.e.,

$$C_t=\Theta\overline{Y}_{NH,t}^{\gamma}\overline{Y}_{NF,t}^{1-\gamma}$$

where $\overline{Y}_{NH,t}=\left[\int_0^1 Y_{NH,t}(u)^{\frac{\theta-1}{\theta}}du\right]^{\frac{\theta}{\theta-1}}$ is a bundle of domestically produced differentiated final goods, and $\overline{Y}_{NF,t}$ is a bundle of foreign produced differentiated final goods. The parameter γ is the share of the household expenditure on domestically produced final goods, $1-\gamma$ is the share of the expenditure on imports, and θ is the elasticity of substitution among the differentiated final goods. The term $\Theta=[\gamma^{\gamma}(1-\gamma)^{1-\gamma}]^{-1}$ is a constant for normalization.

By the household's expenditure minimization problem, the demand function for the final goods are

$$Y_{NH,t}^d(u)=\left[\frac{P_{NH,t}(u)}{\overline{P}_{NH,t}}\right]^{-\theta}\frac{\gamma P_t}{\overline{P}_{NH,t}}C_t$$

$$Y_{NF,t}^d=\frac{(1-\gamma)P_t}{\overline{P}_{NF,t}}C_t$$

where the aggregate price index for the final consumption is $P_t=\overline{P}_{NH,t}^{\gamma}\overline{P}_{NF,t}^{1-\gamma}$, the aggregate price index for the domestic produced final goods is $\overline{P}_{NH,t}=(\int_0^1 P_{NH,t}(\mu)^{1-\theta}d\mu)^{\frac{1}{1-\theta}}$, and the aggregate price index for the foreign produced final goods is $\overline{P}_{NF,t}=T_t\mathscr{E}_tP_{NF,t}^*$. The term $\mathscr{E}_t$ is the price of foreign currency in units of domestic currency, $P_{NF,\,t}^*$ is the exogenous foreign price in foreign currency, and T_t is a uniform tariff on imports. An upper star $*$ is used to denote variables in the foreign country (denominated in the foreign currency).

By the household's utility maximization problem, we obtain the labor supply and Euler Equation, respectively, as

$$\frac{W_t}{P_t}=\frac{V_{N,t}'}{U_{c,t}'} \tag{1}$$

and

$$U_{c,t}'=\beta R_tE_t\left[U_{c,t+1}'\frac{P_t}{P_{t+1}}\right] \tag{2}$$

where $R_t=\frac{1}{E_tD_{t,t+1}}$ is the gross return on a one-period risk-free nominal bond in domestic currency.

Assuming a CRRA utility function, i.e., $U(C_t)=\frac{C_t^{1-\sigma}-1}{1-\sigma}$ and $V(N_t)=\frac{L_t^{1+\psi}}{1+\psi}$, the labor supply (1) and Euler Eq. (2) can be written in log-linearized form as

$$w_t-p_t=\sigma c_t+\psi n_t \tag{3}$$

$$c_t=E_t(c_{t+1})-\frac{1}{\sigma}[i_t-E_t(\pi_{t+1})-\rho] \tag{4}$$

where lower-case letters denote the logarithm of the respective variables, $i_t=R_t-1$ is the nominal interest rate in domestic currency, $\pi_{t+1}=p_{t+1}-p_t$ is the CPI inflation, and $\rho=\beta^{-1}-1$.[6]

We assume that the household has access to a complete set of (both domestic and international) state-contingent securities, and trade in the international asset market before the monetary authority chooses its policy. This timing assumption follows the convention in this literature, and ensures a risk-sharing condition that is independent of monetary policy rules (see the discussion in Senay and Sutherland, 2007, and Matsumura, 2019). Then, the intertemporal marginal rates of substitution must be equalized across countries, i.e.,

$$\beta^t\frac{(C_t^*)^{-\sigma}/P_t^*}{(C_0^*)^{-\sigma}/P_0^*}\Lambda=\beta^tC_t^{-\sigma}\mathscr{E}_tP_t^{-1} \tag{5}$$

where Λ is the marginal utility of initial debt for the domestic household, and the risk-sharing condition implies[7]

$$C_t=\theta^*C_t^*Q_t^{1/\sigma} \tag{6}$$

i.e.,

$$c_t=c_t^*+\sigma^{-1}q_t+\xi \tag{7}$$

where the variable $Q_t=\frac{\mathscr{E}_tP_t^*}{P_t}$ is the real exchange rate, $\theta^*=(\Lambda P_0^*)^{-1/\sigma}/C_0^*$ is a constant (i.e., invariant across policies), and the variable $\xi=\ ln\,\theta^*$. Note that if the asset markets cannot insure across different policies, then θ^* (or Λ) will have to vary across policies.

Engel (2016) criticizes this timing assumption on the asset market and advocates using a balanced trade assumption to replace the risk sharing condition. Since trade imbalance is a key feature of open economies and global supply chains, we choose to retain the complete market assumption and the risk sharing condition in our baseline model. This risk-sharing condition facilitates a comparison of our model with Galí and Monacelli (2005) and De Paoli (2009), where a complete asset market is also assumed.[8]

To acknowledge the Engel's critique, we discuss in Appendix I an alternative setup that features a balanced trade without the risk sharing condition. With a second-order approximation of the non-linear system, we calibrate the model under a set of alternative monetary policy rules, estimate the optimal weights on the variables in each rule, and compute the associated welfare loss. A key finding is that the welfare ranking of the monetary policy rules under the balanced trade assumption is the same as under the assumption of a complete asset market and the risk sharing condition. Based on the appendix, we suggest that the Engel's critique may not be important for our particular research question.

We assume that the foreign consumption follows an $AR(1)$ process, i.e., $c_t^*=\rho_c\cdot c_{t-1}^*+\epsilon_{n,t}$ with $\rho_{c^*}\in(0,1)$ and $\epsilon_{c^*}\sim N(0,{\sigma_{c^*}}^2)$. From the risk-sharing condition (7), given exogenous foreign consumption, there is an increase in consumption if and only if the real exchange rate depreciates. Under the assumption of complete international financial markets, it also implies uncovered interest parity, i.e., $i_t-i_t^*=E_t(\Delta e_{t+1})$, where $e_t=\ ln\,\mathscr{E}_t$.

[6] It is equivalent to write $i_t=-logE_tD_{t,t+1}$ and $\rho=-log\beta$.

[7] Details for deriving Eq. (5) can be found in Matsumura (2019).

[8] Galí and Monacelli (2005) assume the Cole-Obstfeld parameterization, which is a knife-edge case when Λ is exogenous to the monetary policy. This is not true in our case. Instead, we assume an exogenous Λ only to facilitate a comparison with the previous literature.

2.2. Firms

Each final good requires N-stages of production, with a large number of domestic firms producing a unit continuum of differentiated outputs and featuring constant returns to scale at each stage. In the first stage, the production function for good $u \in [0, 1]$ is given by

$$Y_{1H}(u) + Y_{1H}^X(u) = A_1 L_1(u)$$

where A_1 is the productivity in stage 1 and $L_1(u)$ is the quantity of labor employed in the production of good u. The output is either sold at home $Y_{1H}(u)$ or exported abroad $Y_{1H}^X(u)$. The stage-1 output sold at home and its corresponding price are given by $Y_{1H} = \left[\int_0^1 Y_{1H}(u)^{\frac{\theta-1}{\theta}} du\right]^{\frac{\theta}{1-\theta}}$ and $P_{1H} = [\int_0^1 P_{1H}(u)^{1-\theta} du]^{\frac{1}{1-\theta}}$, respectively.

In each subsequent stage, the production needs to use intermediate inputs. The production in stage n (for $n = 2, \ldots, N$) can be viewed as a two-step process. In the first step, a firm purchases all differentiated outputs produced in the previous stage $n-1$ from all global sources and form a bundle of intermediate inputs. Specifically, the intermediate input bundle to be used in the production stage n, i.e., $\overline{Y}_n$, is a bundle of two composites of stage $n-1$ outputs:

$$\overline{Y}_n = \Theta \overline{Y}_{(n-1)H}^{\gamma} \overline{Y}_{(n-1)F}^{1-\gamma}$$

$$\overline{Y}_{(n-1)H} = \left[\int_0^1 Y_{(n-1)H}(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$$

where $Y_{(n-1)H}(j)$ is the amount of good j that is domestically produced in stage $n-1$ and purchased by the firm in stage n, and $\overline{Y}_{(n-1)F}$ is the amounts of composite good that foreign firms produced in stage $n-1$. In the factor market, domestic firms are price takers in purchasing foreign composite goods $\overline{Y}_{(n-1)F}$, and because of the small-open economy setup, the supply of foreign composite goods is perfectly elastic in price.

The aggregate price index for the inputs in stage n is then given by $\overline{P}_n = \overline{P}_{(n-1)H}^{\gamma} \overline{P}_{(n-1)F}^{1-\gamma}$, where $\overline{P}_{(n-1)H} = [\int_0^1 P_{(n-1)H}(u)^{1-\theta} du]^{\frac{1}{1-\theta}}$ and $\overline{P}_{(n-1)F} = T_t \mathscr{E}_t P_{(n-1)F}^*$. The variable $P_{(n-1)F}^*$ is the price of composite goods in foreign currency produced in stage $n-1$ by foreign firms. Note that the output price in stage n satisfies $P_{nH} = \overline{P}_{nH}$ for $\forall n = 1, 2, \ldots, N$.

In the second step, the firm combines the composite intermediate good with labor input to produce an output. The production function for good u in stage n is given by

$$Y_{nH}(u) + Y_{nH}^X(u) = \Theta^* A_n \overline{Y}_n(u)^{\phi} L_n(u)^{1-\phi}$$

where $\Theta^* = [(1-\phi)^{1-\phi}\phi^{\phi}]^{-1}$ is a constant for normalization. We assume the technology in each stage following the $AR(1)$ process $a_{n,t} = \rho_n a_{n,t-1} + \epsilon_{n,t}$ with $a_{n,t} = lnA_{n,t}$ and $\rho_n \in (0,1)$ for $n = 1, 2, \ldots, N$. Note that $\{\epsilon_n\}_{n=1}^N$ are i.i.d. shocks with the same normal distribution, i.e., $\epsilon_n \sim N(0, \sigma_a^2)$.

Since the production of any good in stage n needs a bundle of output from the previous stage as inputs, it captures a feature of a typical input-output table in which the output from all sectors may be used as inputs into the production. In the language of Baldwin and Venables (2013), the entire manufacturing production process follows a combination of a snake and a spider patterns. At a given stage, outputs from the previous stage from all over the world are purchased to form a composite intermediate input, resembling a spider pattern. Going from one stage of production to the next, the process resembles a snake pattern.

By the small-open economy set-up, the foreign demand for domestic output in stage $n = 1, 2, \ldots, N$ is assumed to be

$$Y_{nH}^{Xd}(u) = \left(\frac{P_{nH}(u)}{P_{nH}}\right)^{-\theta} \frac{Y_{nH}^* P_{nH}^* \mathscr{E}_t}{P_{nH}}$$

where Y_{nH}^* is exogenous foreign demand and P_{nH}^* is the price for domestic produced composite goods in foreign currency. This foreign demand function can be derived from the cost minimization problem of a foreign buyer who aggregates the composite of domestic produced goods.

Similarly, the domestic demand function in stage $n = 1, 2, \ldots, N$ is given by

$$Y_{nH}(u) = \left(\frac{P_{nH}(u)}{P_{nH}}\right)^{-\theta} \frac{\overline{Y}_{nH} \overline{P}_{nH}}{P_{nH}}$$

Note that the nominal exchange rate is not a sufficient statistics for import tariffs in a world of multi-stage production. An increase in the nominal exchange rate (i.e., a depreciation of the domestic currency) raises both the input costs and foreign demand for domestic goods simultaneously. In comparison, an increase in import tariffs only affects production cost through higher costs of imported inputs.

2.3. The firm's pricing problem

Firms in each stage of production are price-takers in factor markets, but are monopolistic competitors in their outputs. They are assumed to follow a Calvo pricing rule, and the probability that firms in stage n can adjust prices freely is $1 - \alpha_n, n = 1, \ldots, N$. Then, by the law of large numbers, in each period, a fraction $1 - \alpha_n$ of firms in stage n can adjust prices while the rest of firms have to stay unchanged. For a firm producing good u in stage n, which can set a new price in period t, it chooses price $P_{nH}(u)$ in domestic currency for its product sold both at home and in the foreign market. Its maximization problem becomes

$$max_{P_{nH,t}(u)} E_t \Sigma_{k=t}^{\infty} \alpha_n^{k-t} D_{t,k} \left[(1+\tau) P_{nH,t}(u) - \Psi_{n,k}(u)\right] \left[Y_{nH,k}^d(u) + Y_{nH,k}^{Xd}(u)\right]$$

where τ is the subsidy to firms that corrects the distortion from monopolistic competition, $\Psi_{n,k}(u) = \overline{P}_{n,k}^{\phi} W_k^{1-\phi} / A_{n,k}$ is the nominal unit production cost for $n = 2, \ldots, N$ and $\Psi_{1,k}(u) = W_k / A_{1,k}$ for $n = 1$, $\overline{P}_{n,k}$ is the price for the composite of intermediate input goods at stage n, and $Y_{nH,k}^d(u)$ and $Y_{nH,k}^{Xd}(u)$ denote the output demand from both domestic and foreign market respectively.

The optimal pricing decision is given by

$$P_{nH,t}^o(u) = \frac{\mu}{1+\tau} \frac{E_t \Sigma_{\tau=t}^{\infty} \alpha_n^{\tau-t} D_{t,\tau} \Psi_{n,\tau}(u) \left[Y_{nH,\tau}^d(u) + Y_{nH,k}^{Xd}(u)\right]}{E_t \Sigma_{\tau=t}^{\infty} \alpha_n^{\tau-t} D_{t,\tau} \left[Y_{nH,\tau}^d(u) + Y_{nH,k}^{Xd}(u)\right]}$$

where $\mu = \frac{\theta}{\theta-1}$ is the markup in the market for producing outputs in each stage.[9] To be abstract from the distortion generated by monopolistic competition, a subsidy is imposed such that $1 + \tau = \mu$.

Taking input prices as given, the cost minimization problem for the firms at stage n for $n = 2, \ldots N$ in period t yields a factor demand function as

$$\overline{Y}_{n,t}^d = \phi \frac{\Psi_{n,t}}{\overline{P}_{n,t}} \int_0^1 \left[Y_{nH,t}^d(u) + Y_{nH,t}^{Xd}(u)\right] du \quad (8)$$

$$L_{n,t}^d = (1-\phi) \frac{\Psi_{n,t}}{W_t} \int_0^1 \left[Y_{nH,t}^d(u) + Y_{nH,t}^{Xd}(u)\right] du \quad (9)$$

[9] Since we assume the same elasticity of substitution among differentiated goods across the stages, it also implies the same markup across the stages.

$$\overline{Y}^d_{(n-1)H,t} = \frac{\gamma \overline{P}_{n,t}}{\overline{P}_{(n-1)H,t}} \overline{Y}^d_{n,t} \tag{10}$$

$$\overline{Y}^d_{(n-1)F,t} = \frac{(1-\gamma)\overline{P}_{n,t}}{\overline{P}_{(n-1)F,t}} \overline{Y}^d_{n,t} \tag{11}$$

and

$$Y^d_{(n-1)H,t}(u) = \left(\frac{P_{(n-1)H,t}(u)}{\overline{P}_{(n-1)H,t}}\right)^{-\theta} \overline{Y}^d_{(n-1)H,t} \tag{12}$$

In the first stage of production, the firm's pricing problem is simpler since labor is the only input. Specifically, the optimal pricing decision for a firm in stage 1 is

$$P^o_{1H,t}(u) = \frac{E_t \Sigma^\infty_{\tau=t} \alpha_1^{\tau-t} D_{t,\tau} \Psi_{1,\tau}(u) \left[Y^d_{1,\tau}(u) + Y^{Xd}_{1H,k}(u)\right]}{E_t \Sigma^\infty_{\tau=t} \alpha_1^{\tau-t} D_{t,\tau} \left[Y^d_{1,\tau}(u) + Y^{Xd}_{1H,k}(u)\right]}$$

where $\Psi_{1,\tau}(u) = W_\tau / A_{1,\tau}$ is the unit production cost in stage 1, and the subsidy has been imposed to offset the markup.

Since labor is the only input in the first stage, the labor demand is

$$L^d_{1,t} = \frac{\Psi_{1,t}}{W_t} \int_0^1 \left[Y^d_{1H,t}(u) + Y^{Xd}_{1H,t}(u)\right] du$$

As the goods are symmetric, we drop good index u in the price variable. The aggregate price index for the outputs in stage n, $n = 1, 2, \dots, N$, is thus given by

$$P_{nH,t} = \left[\alpha_n P^{1-\theta}_{nH,t-1} + (1-\alpha_n)\left(P^o_{nH,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{13}$$

2.4. The market clearing conditions and equilibrium definition

Equilibrium definition: given exogenous monetary policy (the rule of nominal interest rate or nominal exchange rate $\{i_t, \mathscr{E}_t\}$) and tariffs $\{T_t\}$, as well as exogenous foreign demand and foreign prices $\{C^*_t, P^*_{nH,t}, P^*_{nF,t}, Y^*_{nH,t}\}^N_{n=1}$, the market equilibrium consists of a set of stochastic processes – $\{C_t, L_t\}$ for domestic households, $\{L^d_{n,t}(u), Y_{nH,t}(u), Y^X_{nH,t}(u), P_{nH,t}(u)\}^N_{n=1}$ for firms $u \in [0, 1]$ and price indices $\{P_{nH,t}\}^N_{n=1}$, and wages and real exchange rate $\{W_t, Q_t\}$, satisfying the following conditions:

1. Taking prices and wages as given, the representative household maximizes its utility.
2. Taking intermediate input goods prices, wages, and all output prices except their own's as given, firms in each stage maximize their profits.
3. The intertemporal trade balance condition holds.
4. The labor market clears, and the goods markets clear in all production stages, i.e.,

$$L_t = \sum_{n=1}^N L^d_t, \quad Y_{nH} = Y^d_{nH}, \quad Y^X_{nH} = Y^{Xd}_{nH}$$

Note that the intertemporal trade condition is derived from a no-Ponzi-game condition in the household's debt, which does not necessarily require trade balance in each period. The case of a balanced trade is discussed in Appendix I.

3. The case of two-stage production

If we assume two stages of production, we can obtain a number of analytical expressions.[10] We now characterize sequentially the steady-state, flexible-price, and sticky-price equilibria. We derive the second-order approximation of the welfare loss function for the sticky price case. With sticky prices, there is misallocation of labor across production stages. Because the terms of trade externality and the labor allocation distortions interact with each other, the real exchange rate and the relative price gaps between the production stages enter the welfare loss function.

The model lends itself well to thought experiments on how a change in openness affects the optimal monetary policy rule. This also facilitates a discussion on how a change in the import tariff affects monetary policy. While we only consider domestic productivity shocks in this section, a broader set of stochastic shocks are considered in the numerical analysis in Section 4.

3.1. The steady-state equilibrium

In the steady state, $A_1 = A_2 = 1$, and foreign variables are kept constant. The price index satisfies $P_{1H} = \overline{P}_{1H} = P_{1H}(u)$ and $P_{2H} = \overline{P}_{2H} = P_{2H}(u)$. By $\overline{P}_2 = \overline{P}^\gamma_{1H} \overline{P}^{1-\gamma}_{1F}$, from Section 2.2, the price indices of domestically produced goods across stages are given by $P_{1H} = W$ and $P_{2H} = W^{1-\phi+\gamma\phi}(T\mathscr{E})^{(1-\gamma)\phi}(P^*_{1F})^{(1-\gamma)\phi}$.

Given $P_{1H}(u) = P_{1H}$ and $P_{2H}(u) = P_{2H}$, the demand for good u in both two stages satisfies $Y^d_{1H}(u) = \overline{Y}^d_{1H}$ and $Y^d_{2H}(u) = \overline{Y}^d_{2H}$. The goods market clearing condition requires $Y_{1H} = \overline{Y}^d_{1H}$ and $Y_{2H} = \overline{Y}^d_{2H}$. Therefore, the factor demand functions are given by

$$\overline{Y}^d_2 = \phi W^{1-\phi}\left(\overline{P}_2\right)^{1-\phi}\left(Y^d_{2H} + Y^{Xd}_{2H}\right)$$

$$L^d_2 = (1-\phi) W^{-\phi} \overline{P}_2^{-\phi}\left(Y^d_{2H} + Y^{Xd}_{2H}\right)$$

$$Y^d_{1H} = \frac{\gamma \overline{P}_2 \overline{Y}^d_2}{\overline{P}_{1H}}$$

and

$$L^d_1 = Y^d_{1H} + Y^X_{1H}$$

where $Y^{Xd}_{2H} = \frac{Y^*_{2H} P^*_{2H} \mathscr{E}}{P_{2H}}$, $Y^{Xd}_{1H} = \frac{Y^*_{1H} P^*_{1H} \mathscr{E}}{P_{1H}}$, $Y^d_{2H} = \gamma C \overline{P}_{2H}^{-(1-\gamma)} \overline{P}_{2F}^{1-\gamma}$, and $\overline{P}_{2F} = T P^*_{2F} \mathscr{E}$. By backward induction, we can obtain the labor demand function in each stage of production.

The equilibrium in the steady state $\{C, L\}$ is then fully characterized by the labor supply Eq. (1), the risk sharing condition (6), and the labor demand function $L^d_1 + L^d_2$ as derived above, where all price indices are a function of W and $\mathscr{E}$. Following Huang and Liu (2005), we set $\psi = 0$ to simplify expressions, which can be justified by indivisible labor (e.g., Hansen, 1985).[11] Then, Eqs. (1) and (6) give

$$w = \sigma c^* + e + p^*$$

$$c = c^* + \frac{1}{\sigma}[e + p^* - p] + \xi$$

By substituting w into $p = \gamma \overline{p}_{2H} + (1-\gamma)\overline{p}_{2F}$, together with $\sigma c = \sigma c^* + e + p^* - p + \sigma\xi$, we obtain an expression of c, which includes neither the domestic price index nor nominal exchange rate. Similarly,

[10] We present the results for the general case of N stages of production in Appendix A.

[11] In the numerical analysis in Section 4, we use a more general value of ψ based on calibrations.

by substituting w into price index, together with $L = L_1^d + L_2^d$, we obtain an expression of steady-state labor l.

3.2. The flexible-price equilibrium

In the flexible-price equilibrium, $\alpha_n = 0$ for $n = 1, 2$. The optimal pricing decision for firms at stage n becomes $P^o_{nH,t} = \Psi_{n,t}$ and thus $P_{nH,t} = \overline{P}_{nH,t} = P^o_{nH,t}$.

With $\overline{P}_{2,t} = \overline{P}_{1H,t}^{\gamma}\overline{P}_{1F,t}^{1-\gamma}$, stage-specific prices indices are given as $P_{1H,t} = W_t/A_{1,t}$ and $P_{2H,t} = W^{1-\phi+\gamma\phi}(T_t\mathscr{E}_t)^{(1-\gamma)\phi}(P^*_{1F})^{(1-\gamma)\phi}A_{1,t}^{-\gamma\phi}A_{2,t}^{-1}$. The aggregate price for final consumption goods is $P_t = (P_{2H,t})^{\gamma}(T_tP^*_{2F,t}\mathscr{E}_t)^{1-\gamma}$, in which we have plugged the expression of $\overline{P}_{2F,t}$.

Similar to the analysis in the steady-state equilibrium, we have $Y^d_{1H,t}(u) = \overline{Y}^d_{1H,t}$, $Y^d_{2H,t}(u) = \overline{Y}^d_{2H,t}$, $Y_{1H,t} = \overline{Y}^d_{1H,t}$ and $Y_{2H,t} = \overline{Y}^d_{2H,t}$. The factor demand functions are given by

$$\overline{Y}^d_{2,t} = \phi\frac{W_t^{1-\phi}\left(\overline{P}_{2,t}\right)^{1-\phi}}{A_{2,t}}\left(Y^d_{2H,t} + Y^{Xd}_{2H,t}\right)$$

$$L^d_{2,t} = (1-\phi)\frac{W_t^{1-\phi}\left(\overline{P}_{2,t}\right)^{1-\phi}}{A_{2,t}}\left(Y^d_{2H,t} + Y^{Xd}_{2H,t}\right)$$

$$Y^d_{1H,t} = \frac{\gamma\overline{P}_{2,t}\overline{Y}^d_{2,t}}{\overline{P}_{1H,t}}$$

and

$$L^d_{1,t} = Y^d_{1H,t} + Y^X_{1H,t}$$

where $Y^X_{2H,t} = \dfrac{Y^*_{2H,t}P^*_{2H,t}\mathscr{E}_t}{P_{2H,t}}$, $Y^X_{1H,t} = \dfrac{Y^*_{1H,t}P^*_{1H,t}\mathscr{E}_t}{P_{1H,t}}$, $Y^d_{2H,t} = \gamma C\overline{P}_{2H,t}^{-(1-\gamma)}\overline{P}_{2F,t}^{1-\gamma}$, and $\overline{P}_{2F,t} = T_tP^*_{2F,t}\mathscr{E}_t$.

Similar to the steady-state equilibrium, the flexible-price equilibrium $\{C_t, L_t\}$ are then fully characterized by the labor supply Eq. (1), the risk sharing condition (6), and the labor demand function $L_1^d + L_2^d$ as derived above. With the assumption of $\psi = 0$, the Eqs. (1) and (6) again give

$$w_t^f = \sigma c_t^* + e_t^f + p_t^*$$

$$c_t^f = c_t^* + \frac{1}{\sigma}\left[e_t^f + p_t^* - p_t^f\right] + \xi$$

where we denote the endogenous variables under flexible price equilibrium with an upper symbol f.

By substituting w_t^f into price index, we obtain the expressions of c_t^f and l_t^f, which does not include domestic price index or nominal exchange rate. Note that, by denoting $t_t = lnT_t$, we have the expression of CPI index p_t^f as

$$p_t^f = \gamma\left[(1-\phi+\gamma\phi)w_t^f + (1-\gamma)\phi\left(e_t^f + t_t\right) + (1-\gamma)\phi p^*_{1F,t} - \gamma\phi a_{1,t} - a_{2,t}\right]$$
$$+(1-\gamma)\left(e_t^f + t_t + p^*_{2F,t}\right)$$

By substituting p_t^f into the risk-sharing condition, we obtain the natural rate of interest rate as

$$\overline{r}r = \rho + \sigma E\left(c^f_{t+1} - c^f_t\right)$$
$$= \rho + \gamma\left[\gamma\phi\rho_1\Delta a_{1,t} + \rho_2\Delta a_{2,t}\right]$$

where we treat exogenous foreign variables and import tariff as constant.

3.3. The sticky-price equilibrium

We now derive the New Keynesian Phillips curves for each stage as a function of the relative price gap and output gap, and characterize the equilibrium with sticky prices. Similar to the derivation in Galí (2015), the Phillips curve for each stage is given by

$$\pi_{1H,t} = \beta E_t\pi_{1H,t+1} + \lambda_1\hat{y}_{1,t}$$

$$\pi_{2H,t} = \beta E_t\pi_{2H,t+1} + \lambda_2\hat{y}_{2,t}$$

where $\lambda_n = \dfrac{(1-\beta\alpha_n)(1-\alpha_n)}{\alpha_n}$ for $n = 1, 2$ and $\hat{y}_n$ is the log-derivation of real marginal cost from steady-state equilibrium, i.e.,

$$\hat{y}_{n,t} = \ln(\Psi_{n,t}/P_{nH,t}) - \ln(\Psi_n/P_{nH})$$

Since Ψ_n and P_{nH} are the marginal cost and aggregate price in stage n under steady-state equilibrium, we have

$$\hat{y}_{1,t} = \sigma\hat{c}_t + \frac{1-\gamma}{\gamma}\hat{q}_t - \hat{g}_{2H,t} - a_{1,t}$$

$$\hat{y}_{2,t} = \gamma\phi\hat{g}_{2H,t} + \frac{1-\gamma}{\gamma}\hat{q}_t + (1-\phi)\sigma\hat{c}_t - a_{2,t}$$

where $\hat{g}_{2H,t}$ is the log-deviation of relative price gap between stage-2 output price with respect to stage-1 output price from the steady-state equilibrium, i.e., $\hat{g}_{2H,t} = \ln(P_{1H,t}/P_{2H,t}) - ln(P_{1H}/P_{2H})$.[12] In terms of notation, we use the variable with a hat to denote deviation from the steady-state equilibrium, and use a tilde to denote the deviation from the flexible-price equilibrium.

After log-linearizing the Euler equation of the household around the steady state and subtracting the steady-state IS curve, we obtain the IS curve as

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}\left[\hat{i}_t - E_t(\pi_{t+1})\right]$$

The aggregate inflation π_t (CPI index) can be written as

$$\pi_t = \pi_{2H,t} + \frac{1-\gamma}{\gamma}\Delta\hat{q}_t$$

The derivation of the aggregate inflation can be found in Appendix B.

The law of motion for the relative price gap between stage 1 and stage 2 is characterized by

$$\hat{g}_{2H,t} = \hat{g}_{2H,t-1} + \pi_{1H,t} - \pi_{2H,t}$$

The above equations together with the risk-sharing condition (7) fully characterize the sticky-price equilibrium.

3.4. A utility-based welfare loss function for optimal monetary policy

We assume that the central bank aims to maximize the household's utility, and represent its objective function by a second-order approximation. This follows the approach of Rotemberg and Woodford (1999), Benigno and Woodford (2006), and Galí (2015). Due to simultaneous presence of openness and multiple stages of production, the first-order terms do not cancel each others out, unlike in the standard literature. This means that the welfare loss function in

[12] The proofs for deriving the expressions for $\hat{y}_1$ and $\hat{y}_2$ are in Appendix A.3, which characterizes the sticky-price equilibrium with N stages of production.

our setting includes inflation for each stage of production, the relative price gap across production stages, as well as the real exchange rate and output gap. The flexible-price equilibrium is, in general, not Pareto-optimal. Only in the limit case of a closed economy would the first-order terms cancel out and the welfare loss function is left only with the second-order terms as shown in Appendix F. In such a case, both the steady-state equilibrium and the flexible-price equilibrium are Pareto-efficient.

Since labor is present in all stages of production and prices are sticky, there is labor misallocation across production stages, which reduces the utility of the household. There is also a terms-of-trade externality since some of the domestically produced intermediate goods are exported and some imported intermediate inputs are used in domestic production.

The household's utility function is given by

$$E\Sigma_{t=0}^{\infty}\beta^t[U(C_t)-V(L_t)]$$

where $U(C_t)=\frac{C_t^{1-\sigma}-1}{1-\sigma}$ and $V(L_t)=\frac{L_t^{1+\psi}}{1+\psi}$.

A second-order Taylor expansion around the steady state (C,L) for the period utility of consumption gives

$$U(C_t)-U=U_cC\left(\hat{c}_t+\frac{1-\sigma}{2}\hat{c}_t^2\right)+t.i.p.$$

where $\hat{c}_t$ denotes the log-deviation of consumption from steady state and $t.\ i.\ p.$ stands for "terms independent of policy" following Woodford (2003).

By the labor market clearing condition, we obtain the second-order Taylor expansion around the steady state for the period utility of employment, i.e., $V(L_t)$, as

$$V(L_t)-V=V_LL\left\{\sum_{n=1}^{2}\frac{L_n}{L}\left[\hat{l}_{n,t}+\frac{1}{2}\hat{l}_{n,t}^2\right]\right\}+t.i.p.$$

where L_n/L is the share of labor in stage n in total labor in the steady state, as described in Section 3.1, with the assumption of $\psi=0$.[13]

It is useful to rewrite the employment gap in the two production stages in terms of the output gap and the relative price gap:

$$\hat{l}_{1,t}=[-\bar{a}_1\sigma\phi+\bar{a}_1\bar{a}_2+\bar{a}_1\sigma]\hat{c}_t+[\bar{a}_1\phi\gamma-1]\hat{g}_{2H,t}$$
$$+\frac{1+\bar{a}_1-\bar{a}_1\gamma-\bar{a}_1\bar{a}_2\gamma}{\gamma}\hat{q}_t+\bar{a}_1d_{2,t}+d_{1,t}-a_{1,t}-\bar{a}_1a_{2,t}$$

$$\hat{l}_{2,t}=(\bar{a}_2-\phi\sigma)\hat{c}_t+\phi\gamma\hat{g}_{2H,t}+\frac{1-\bar{a}_2\gamma}{\gamma}\hat{q}_t+d_{2,t}-a_{2,t}$$

where $d_{n,t}=\ \ln\left(\int_0^1\left(\frac{P_{nH,t}(u)}{P_{nH,t}}\right)^{-\theta}du\right)$ for $n=1,2$ measures the price dispersion in stage n, $\bar{a}_1=\frac{Y_{1H}^d}{Y_{1H}^d+Y_{1H}^X}$ and $\bar{a}_2=\frac{Y_{2H}^d}{Y_{2H}^d+Y_{2H}^X}$ are the shares of goods sold, respectively, to the domestic market in stages 1 and 2 in the steady state. Details can be found in Appendix C.

Following Galí (2015), up to a second-order approximation around the steady state, the price dispersion term $d_{n,t}$ for $n=1,2$ can be written as

$$d_{n,t}=\frac{\theta}{2}\int_0^1\left[p_{nH,t}(i)-p_{nH,t}\right]^2di\equiv\frac{\theta}{2}\,var\{p_{nH,t}(i)\}$$

By Woodford (2003), the price dispersion can be re-written as a function of inflation in each stage of production, i.e.,

$$\sum_{t=0}^{\infty}\beta^t\ var\{p_{nH,t}(i)\}=\lambda_n^{-1}\sum_{t=0}^{\infty}\beta^t\pi_{nH,t}^2+t.i.p.$$

We substitute $\hat{l}_{n,t}$ and $d_{n,\,t}$ into the period utility of employment. Since the total labor income of households are given by $WL=\frac{PC\gamma}{\bar{a}_2}(1-\phi)+\phi\frac{PC\gamma}{\bar{a}_2}\frac{\gamma}{\bar{a}_1}$, the steady state equilibrium implies $WL=PC(1-\Phi)$, where $1-\Phi=\frac{\gamma}{\bar{a}_2}(1-\phi)+\phi\frac{\gamma^2}{\bar{a}_2\bar{a}_1}$, and thus $U_cC=V_LL(1-\Phi)$.[14] In addition, in the steady state, the labor shares in the two stages are given, respectively, by $L_1/L=\frac{\gamma\phi}{\gamma\phi+(1-\phi)\bar{a}_1}$ and $L_2/L=\frac{(1-\phi)\bar{a}_1}{\gamma\phi+(1-\phi)\bar{a}_1}$.

By summing up $U(C_t)-U$ and $V(L_t)-V$, the household's welfare loss as a fraction of the steady state consumption is given by

$$\begin{aligned}W&=E_0\sum_{t=0}^{\infty}\beta^t\frac{U(C_t)-V(L_t)-(U-V)}{U_cC}\\&=E_0\sum_{t=0}^{\infty}\beta^t\hat{c}_t-\sum_{n=1}^{2}(1-\Phi)\frac{L_n}{L}\hat{h}_{n,t}-\frac{1}{2}\left\{-(1-\sigma)\hat{c}_t^2+(1-\Phi)\left[\frac{L_1}{L}\left(\hat{h}_{1,t}-a_{1,t}-\bar{a}_1a_{2,t}\right)^2\right.\right.\\&\left.\left.+\frac{L_2}{L}\left(\hat{h}_{2,t}-a_{2,t}\right)^2+\frac{L_1}{L}\theta\lambda_1^{-1}\pi_{1H,t}^2+\left(\frac{L_1}{L}\bar{a}_1+\frac{L_2}{L}\right)\theta\lambda_2^{-1}\pi_{2H,t}^2\right]\right\}+t.i.p.\end{aligned}\tag{14}$$

where

$$\hat{h}_{1,t}=(-\bar{a}_1\sigma\phi+\bar{a}_1\bar{a}_2+\bar{a}_1\sigma)\hat{c}_t+(\bar{a}_1\phi\gamma-1)\hat{g}_{2H,t}+\frac{1+\bar{a}_1-\bar{a}_1\gamma-\bar{a}_1\bar{a}_2\gamma}{\gamma}\hat{q}_t$$

$$\hat{h}_{2,t}=(\bar{a}_2-\phi\sigma)\hat{c}_t+\phi\gamma\hat{g}_{2H,t}+\frac{1-\bar{a}_2\gamma}{\gamma}\hat{q}_t$$

The first-order terms can be eliminated by approximating the equilibrium conditions specified in Section 3.3 to a second-order expansion using the approach developed by Sutherland (2002) and Benigno and Woodford (2006). Though we do not present an explicit expression of the welfare loss function purely in second-order terms due to the complexity arising from the multi-stage production, the numerical analysis in Section 4 approximates the full *nonlinear* equilibrium in the second-order expansion.

Our setup nests several models in the existing literature as special cases. In particular, if we shut down economic openness, and assume $N=2,\gamma=1$, and $\bar{a}_1=\bar{a}_2=1$, the expression (14) reproduces the welfare loss function in Huang and Liu (2005). Alternatively, if we maintain the small-open economy structure, but assume one stage of production $(N=1)$, and $\bar{a}_1=\gamma$, and additionally impose symmetry in the foreign country, the expression (14) reproduces the welfare loss function in Galí and Monacelli (2005) and De Paoli (2009).[15]

[13] As specified for the steady-state variables in Section 3.1, we have $\frac{L_1}{L}=\frac{\gamma\phi}{\gamma\phi+(1-\phi)\bar{a}_1}$ and $\frac{L_2}{L}=1-\frac{L_1}{L}$, where $\bar{a}_1=\frac{Y_{1H}^d}{Y_{1H}^d+Y_{1H}^X}$ and $\bar{a}_2=\frac{Y_{2H}^d}{Y_{2H}^d+Y_{2H}^X}$ are the share of goods sold at home, respectively, in stages 1 and 2.

[14] In the small-open economy New Keynesian literature, Φ is normally assumed to be zero due to the symmetry assumption across countries in a two-country structure model, e.g., Faia and Monacelli (2008), De Paoli (2009), or in a model of continuum of small countries, e.g., Galí and Monacelli (2005).

[15] In the case of one-stage production, the results in Galí and Monacelli (2005) and De Paoli (2009) are reproduced by also recognizing that the foreign demand now is foreign final demand. It is also worth noting that De Paoli (2009) allows for a general parameterization of elasticity of substitution regarding foreign goods and domestic goods, while we assume the elasticity to be one by taking a Cobb-Douglas form.

To shed light on the role of the length of production chain in affecting the welfare, we derive analytical results in the case of a closed-economy (see Appendix F). In this case, both the steady-state equilibrium and the flexible-price equilibrium are Pareto-efficient, and the welfare loss function can be expressed in second-order terms. In particular, the stage-specific inflation terms have a direct impact, given by the expression of $\sum_{n=1}^{N}\theta\phi^{N-n}\lambda_n^{-1}\pi_{n,t}^2$, in the welfare loss function.

Two features of these terms deserve special attention. First, for a fixed number of production stages N, assuming the same price stickiness in all stages, the coefficients before inflation in the downstream stages are larger than those in the upstream stages. Second, holding the downstream sectors constant, as one adds more upstream stages, the final stage inflation (i.e., CPI) becomes less important in the welfare loss function, while the inflation rates in the upstream stages as a whole become more important. In other words, as the production chain becomes longer, the central bank needs to care more about the inflation rates in the intermediate stages but less about the final stage inflation.

3.5. Discussion on the welfare loss function

There are two distortions in the model, i.e., the labor allocation distortion (caused by sticky prices along the production chain) and the terms of trade externality. Those terms measuring stage-specific unemployment gaps $\hat{h}_{1,t}$ and $\hat{h}_{2,t}$ show up in welfare loss function because of sticky prices and misallocation of labor across production stage. The real exchange rate $\hat{q}_t$ appears due to the terms-of-trade externality.

In an open economy with a finite elasticity of foreign demand for export, the social planner wishes to exploit a domestic monopoly power in trade. This gives rise to a terms of trade effect. As the real exchange rate $\hat{q}_t$ and the relative price gap between production stage, $\hat{g}_{2H,t}$, jointly enter $\hat{h}_{1,t}$ and $\hat{h}_{2,t}$, we see an interaction between the labor allocation distortion and the terms of trade distortion. This interaction suggests that the monetary policy discussion is not a simple sum of the results from an open-economy with one stage of production and a closed economy with two stages of production.

The second-order terms in the welfare loss function consist of three parts: (a) a consumption gap, and stage-specific unemployment gaps which can be written in terms of the consumption gap, (b) separate inflation terms for each production stage, and the relative price gap between production stages, and (c) the real exchange rate. The consumption gap is connected with the output gap and real exchange rate via $\hat{y} = \gamma\hat{c} + \frac{1-\gamma^2}{\gamma}\hat{q}$.

The welfare loss function indicates that targeting CPI and PPI is not sufficient. Instead, the central bank needs to pay attention to stage-specific inflation terms along the production process as well as the price gap across the production stages. These terms will become more important as the economy becomes more open or when the number of production stages increase. The last point is elaborated in Appendix F.2 when we consider the case of N-stage production in a closed economy.

3.6. Discussion on value chains and price stickiness

A key feature studied in this paper is a *vertical* structure of production chain or value chain. To highlight the role of vertical structure and clarify its differences with a horizontal production structure, let us consider how a shock propagates along the production chain. The key logic was first pointed out by Huang and Liu (2001) in a closed-economy setting. The same carries over to an open-economy setting.

Let us consider a shock to the nominal wage, which may be caused by an exogenous monetary shock, and focus on a partial equilibrium in which the exchange rate is taken as fixed for simplicity. Since labor is the only input in the first stage of production, the marginal cost of the first-stage production changes immediately following the wage shock, but only a fraction of the firms in the first stage reset their prices due to price rigidity. For this reason, the first-stage output prices, which are the input prices for the second stage, only partially reflect the true change in the labor cost.

For firms in the second stage production, since they use both labor and intermediate goods for production, their marginal cost experiences a smaller change compared to the first-stage output prices. The firms in the second stage thus have less incentive to adjust their prices even though they have the opportunity to do so. The second-stage output prices deviate from those in a flexible price equilibrium more than the first-stage output prices.

In general, when there are N-stages of production, the output prices of more downstream stages are more sluggish than their more upstream counterparts. In this sense, the vertical production structure creates endogenously exacerbating price rigidity moving from upstream to downstream stages along the production chain. This feature does not exist for a horizontal production structure.

3.7. Effects of a higher import tariff

Motivated by a recent rise in international trade tensions, we study how a change in the trade policy, which alters the cost of supply chain trade, may affect the design of the monetary policy. We compare a high-tariff case with a low-tariff case. In each case, the import tariff affects the welfare loss function through its impact on the steady-state shares of the domestic demand in the total demand for domestically produced goods in the two stages of production, i.e., $\bar{a}_1$ and $\bar{a}_2$.

It can be shown that

$$\frac{\bar{a}_2}{1-\bar{a}_2} = f_2(*) \cdot T^{(1-\gamma)(1+\phi\gamma)\left(1-\frac{1}{\sigma}\right)}$$

and

$$\frac{\bar{a}_1}{1-\bar{a}_1} = f_2(*) \cdot \frac{1}{1-\bar{a}_2}$$

where $f_1(*)$ and $f_2(*)$ are functions of exogenous foreign variables. The explicit expression of $f_1(*)$ and $f_2(*)$ can be found in Appendix D. We proceed with the following proposition.[16]

Proposition 1. If the relative risk aversion $\sigma = 1$, a higher import tariff does not affect the steady-state allocation, i.e., $\frac{\partial\bar{a}_1}{\partial T} = \frac{\partial\bar{a}_2}{\partial T} = 0$ and $\frac{\partial L_1/L}{\partial T} = \frac{\partial L_2/L}{\partial T} = 0$; if $\sigma > 1$, a higher import tariff will lead to a higher share of domestic demand for domestically produced goods, i.e., $\frac{\partial\bar{a}_1}{\partial T}$, $\frac{\partial\bar{a}_2}{\partial T} > 0$, and the labor share in the upstream production relative to the downstream decreases, i.e., $\frac{\partial L_1/L}{\partial T} < 0$ and $\frac{\partial L_2/L}{\partial T} > 0$.

4. Comparing monetary policy rules

We consider a family of simple monetary policy rules. As discussed in Section 3, the first-order approximation for the equilibrium conditions is not enough for welfare analysis. We thus estimate the general *nonlinear* model specified in Section 2 with $N = 2$ and approximate the equilibrium by the second order expansion (of both the constraints as well as the welfare function). We relax the assumption of $\psi = 0$ and include a broader set of stochastic shocks, i.e., stage-specific

[16] We assume that $\gamma < 1$, i.e., the share of import is not zero.

Table 1
Parameter calibration.

Parameter	Name	Value	Notes
β	Subjective discount factor	0.99	4% annual interest rate with a quarterly model
σ	Inverse of intertemporal elasticity of substitution	2	Standard value in literature, e.g., Arellano (2008)
α_1,α_2	Parameter in Calvo pricing	0.66	An average length of price contract of 3 quarters
γ	Share of goods purchased in domestic market	0.6	Implying 40% import share of GDP
θ	Elasticity of substitution in consumption bundle	10	Following Benigno and Woodford (2005)
ϕ	Share of intermediate goods in production	0.6	Following Huang and Liu (2005)
$\overline{a}_1, \overline{a}_2$	Share of goods selling to domestic market	0.7	Implying 30% export share of GDP
ρ_a	Persistency of productivity shock	0.66	Following Galí and Monacelli (2005), De Paoli (2009)
σ_a	Standard deviation of productivity shock	0.0071	Following Galí and Monacelli (2005), De Paoli (2009)
ρ_{C^*}	Persistency of foreign consumption shock	0.66	Following De Paoli (2009)
σ_{C^*}	Standard deviation of foreign consumption shock	0.0129	Following De Paoli (2009)

productivity shocks and shocks on foreign consumption (which are the two types of shocks most commonly studied in the literature).

We consider the following set of policy rules: (a) a classic Taylor (1993) rule that is based on CPI inflation (and output gap)[17]; (b) a Galí-Monacelli (2005) rule that replaces CPI inflation with PPI inflation; (c) a rule that targets separate inflation terms for each production stage (i.e., stage-specific producer price indices); (d) combinations of the above with the real exchange rate; and (e) an exchange rate peg. For each rule, we examine both the case with imposed coefficients as specified in the literature (such as 1.5 and 0.5 on CPI inflation and output gap in the classic Taylor rule) and optimally estimated coefficients.

Since global supply chains have been gaining importance over the last two decades but face disruptions by recent tariff wars, we conduct comparative statics exercises on how the optimal weight on upstream inflation relative to the final stage inflation changes in response to changes in an economy's openness. Specifically, we consider a range of openness parameter measured by the export share in sales. For each scenario, we estimate the optimal weights on the production-stage-specific inflation terms as well as on other variables. We then look at how the relative weights evolve as the degree of openness changes.

Asymmetric price stickiness along the production chain appears to be empirically relevant. Cornille and Dossche (2008) and Nakamura and Steinsson (2008) both suggest that the price contracts in more upstream production stages tend to have a shorter duration than those in the finished product sectors. Gong et al. (2016) argue that different degrees of price stickiness in different stages would affect which price index (i.e., CPI, final-goods-based PPI, or intermediate-goods-based PPI) should be included in a simple monetary policy rule.[18]

4.1. Model parameters

We begin with the calibration of parameters for the baseline model. Each period in the model corresponds to a quarter. Following Galí and Monacelli (2005) and De Paoli (2009), the model economy is meant to resemble Canada in some key dimensions. The calibrated parameters are summarized in Table 1.

The subjective discount factor is set to be $\beta = 0.99$, which implies a 4% annual real interest rate in the steady state. Following Arellano (2008) and De Paoli (2009), the inverse of intertemporal elasticity of substitution is set to be $\sigma = 2$. The parameter in Calvo pricing in both production stages is set to be $\alpha_1 = \alpha_2 = 0.66$, implying an average contract duration of 3 quarters. Following Benigno and Woodford (2005), the elasticity of substitution in the consumption bundle is set to be $\theta = 10$. Consistent with Huang and Liu (2005), we set the share of intermediate goods in production to be $\phi = 0.6$.

We set the shares of goods sold to the domestic markets in both stages to be $\overline{a}_1 = \overline{a}_2 = 0.7$, implying a 30% export share of GDP (approximately the level observed for Canada since 2010). The parameters $\overline{a}_1, \overline{a}_2$ are the sufficient statistics for the (exogenous) foreign demand for output in the two production stages. Following Galí and Monacelli (2005), the process of productivity shock is set to follow an AR(1) process with persistence parameter $\rho_a = 0.66$ and standard deviation $\sigma_a = 0.0071$, which is calibrated from Canada data. Following De Paoli (2009), the foreign consumption shock is set to an AR(1) process with persistence $\rho_{C^*} = 0.66$ and standard deviation $\sigma_{C^*} = 0.0129$. We normalize the import tariff in the baseline numerical exercise to be $T = 1$ (implying a zero tariff).

4.2. Welfare losses

The numerical estimation is conducted based on the general nonlinear model specified in Section 2 with $N = 2$. The equilibrium is estimated up to second order approximation (for both the constraints and the welfare loss function). We define the welfare loss χ in percentage term relative to the steady-state consumption, i.e.,

$$E\Sigma_{t=0}^{\infty}\beta^t\left[\frac{[C(1-\chi)]^{1-\sigma}-1}{1-\sigma}-\frac{L^{1+\psi}}{1+\psi}\right]=V^a$$

where C and L are steady-state consumption and employment, and V^a is the welfare estimated from a given policy rule.

The aggregated PPI index is a sales-weighted average of producer prices index:

$$\pi_{PPI}=(1-\omega)\pi_{1H}+\omega\pi_{2H}$$

where $\omega = \dfrac{P_{1h}(Y_{1h}+Y_{1h}^X)}{P_{1h}(Y_{1h}+Y_{1h}^X)+P_{2h}(Y_{2h}+Y_{2h}^X)}$ is the relative sales-weight in the upstream production stage.

We assume that neither the real marginal cost by production stage nor the relative price gap across production stages can be observed by the central bank. So they do not enter any monetary policy rule. Within the family of simple rules, the best that the central bank can do is to make the interest rate a function of the upstream producer price inflation, the final stage producer price inflation, change in the real exchange rate, the output gap, and one-period lagged interest rate. Since the PPI inflation is a sales-weighted average of the first two terms, and the CPI inflation is a linear combination of the second and the third terms, there is no need to include these terms separately. We estimate the optimal coefficients on these variables, label this best possible rule as Policy Rule 1, and normalize its welfare loss to one. Table 2 reports the optimally estimated coefficients for ten different monetary policy rules. The welfare loss for each rule is expressed as relative to that under Policy Rule 1.[19] "Peg" in the table indicates a nominal exchange rate peg.

[17] Henderson and McKibbon (1993) have proposed a similar rule.

[18] Instead of including all stage-specific price indices in a simple monetary rule, Gong et al. (2016) consider a CPI-based Taylor rule, a final-goods PPI-based rule, and a intermediate-goods PPI-based rule. In other words, their rules always include one inflation index plus an output gap.

[19] The welfare loss for P1 in Table 2 in term of steady-state consumption is 0.00319%.

Table 2
Optimal alternative simple rules of monetary policy.

	π_{1H}	π_{2H}	π_{PPI}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{i}_{t-1}$	Welfare loss
P1	6.3861	9.8675			0.7570	−1.5549	0.2048	1
P2				5.1882	0.0006		0.0215	1.809
P3			9.9999		0.1000		1.0441	1.031
P4			9.9888	0.0009	0.0004		0.8085	1.022
P5			9.1138	0.0747	0.1697	−0.6933	0.1432	1.003
P6	5.2870	9.9965			0.0001		0.6948	1.009
P7	4.2548				0.0000		0.8757	1.793
P8		2.6975			0.0003		0.8327	1.257
P9	5.2741	9.9825					0.7004	1.009
Peg								2.730

Notes: PPI index (sales-weighted): $\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$ with $\omega = \dfrac{P_{1h}(Y_{1h}+Y_{1h}^X)}{P_{1h}(Y_{1h}+Y_{1h}^X)+P_{2h}(Y_{2h}+Y_{2h}^X)}$.
CPI index: $\pi_{CPI,t} = \pi_t$.

Table 3
Alternative simple rules of monetary policy in literature.

	π_{1H}	π_{2H}	π_{CPI}	$\hat{c}$	$\hat{i}_{t-1}$	Welfare loss	Notes
P1			1.5	0.5		5.862	Taylor (1993)
P2	1.42	1.68		0.04	1.12	1.166	Huang and Liu (2005)
P3			1.5			2.661	Galí and Monacelli (2005) – CPI based
P4		1.5				3.843	Galí and Monacelli (2005) – PPI based

A classic Taylor rule that targets only CPI inflation and output gap (Policy Rule 2) does terribly in this economy. The welfare loss is 80% higher than Policy Rule 1. The Galí and Monacelli (2005) rule that replaces the CPI inflation with PPI inflation (Policy Rule 3) represents a significant improvement over the classic Taylor Rule in terms of a much smaller welfare loss. Still, the Galí-Monacelli rule is not as good as Policy Rule 1. That is because, with the input-output linkages across production stages, the optimal weights on the upstream sector and final stage inflation terms in Policy Rule 1 are not proportional to the relative sales of the two sectors. Including both PPI and CPI inflation (Policy Rule 4) yields a small improvement over Policy Rule 3 (but a larger improvement over Policy Rule 2). Adding the real exchange rate to Rule 4 (Policy 5) produces more noticeable improvement over Rules 2, 3, or 4. Still, Policy Rule 1 dominates Policy Rule 5.

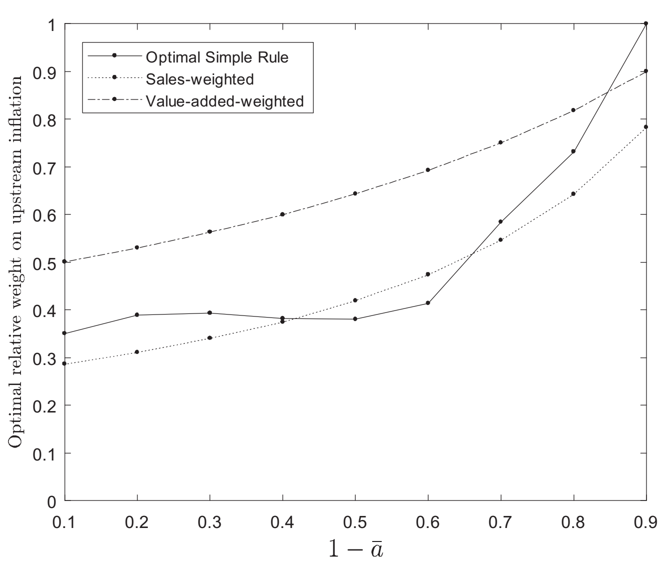

Fig. 2. Relative weight of upstream inflation index in optimal simple rule with respect to country openness.

Policy Rules 6–9 suggest that inflation measures in both the upstream stage and the final stage contain important information. Dropping either one of them from a monetary policy rule could lead to a significant increase in the welfare loss. A nominal exchange rate peg (Policy Rule 10) yields a welfare loss that is 173% higher than Policy Rule 1, which makes an exchange rate peg the worst option among the ten rules considered.

To summarize, the best simple rule would target separate producer price inflation in different stages of production and the real exchange rate (as well as the output gap). If we have to choose among aggregate price indicators, PPI targeting is superior to CPI targeting. In fact, at least with our parameter values, including the PPI inflation moves one not too far from the best simple rule.

For each type of policy rule, besides optimally estimated coefficients, we also evaluate a version where the coefficients are imposed using the values suggested in the literature. Table 3 reports the welfare performance proposed by the original Taylor calibration (i.e., Taylor, 1993), alternative rules adopted in Galí and Monacelli (2005), and in Huang and Liu (2005). The welfare losses in the table are still reported as relative to that under Policy Rule 1 in Table 2. Evidently, simple monetary policy rules that target aggregate PPI, or stage-specific producer indices, outperform those targeting just the CPI index.

4.3. Comparative statics: effects of openness and intermediate goods share

A country's openness and share of intermediate goods in production are the two most important features of global supply chains. In order to study the role of these two factors in optimal simple rules, we conduct comparative statics on how the relative optimal weight on upstream sector inflation changes with respect to these two parameters.

We calibrate foreign demand in both production stages such that the shares of exports in the steady state are the same in both stages ($1-\bar{a}_1 = 1-\bar{a}_2 \equiv 1-\bar{a}$), and both vary from 10% to 90%.

Fig. 2 plots the estimated optimal weight on the upstream sector inflation relative to the sum of the coefficients for the two inflation rates in the two stages, as a function of the openness (measured by the export share, assumed to be common in both stages of production). As shown by the solid line, the optimal relative weight on the upstream sector inflation generally rises as an economy becomes more open

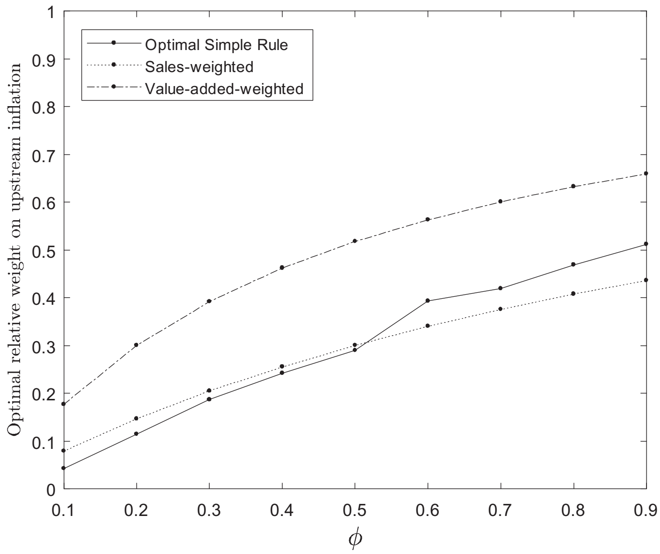

Fig. 3. Relative weight of upstream inflation index in optimal simple rule with respect to intermediate goods share.

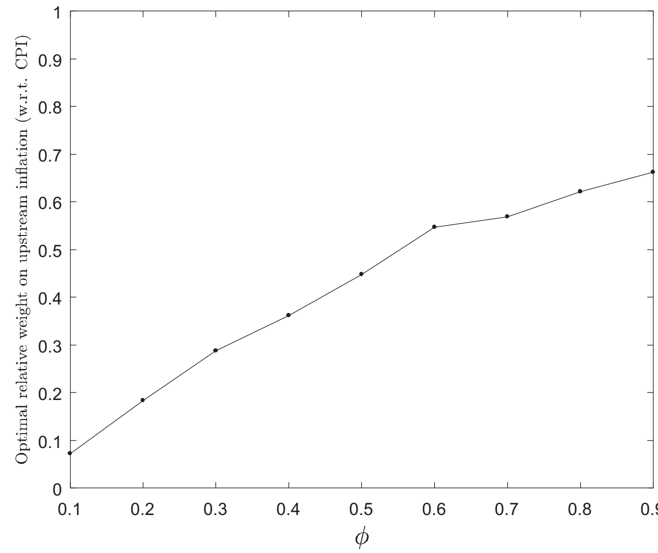

Fig. 4. Ratio of the weight on upstream inflation index versus CPI with respect to intermediate goods share.

(although the increase is not strictly monotonic). This is especially true when the economy evolves from median open to very open (e.g., an increase in openness from 0.6 to 0.9).

The intuition for this result has to do with the vertical production structure. From Section 3.4, the labor shares in the upstream and downstream stages in the steady state are given, respectively, by $L_1/L = \frac{\gamma\phi}{\gamma\phi + (1-\phi)\bar{a}}$ and $L_2/L = \frac{(1-\phi)\bar{a}}{\gamma\phi + (1-\phi)\bar{a}}$. With greater openness (i.e., a smaller $\bar{a}$), a higher share of total employment takes place in the upstream stage. It is therefore sensible to increase the weight on the upstream inflation relative to the downstream inflation in the monetary policy rule. One can infer that, under the classic Taylor rule (which puts zero weight on the upstream sector inflation), the welfare loss would have grown as the economy becomes more open.

In the same graph, we also plot the relative sales weight of the two sectors (the thin dotted line) and the relative value-added weight (the dashed dotted line), respectively. It is clear that the optimal weights on the stage-specific producer inflation are not proportional to either sales or value added of the sectors. This means that targeting the aggregate PPI inflation cannot achieve the same level of welfare as targeting production stage-specific producer price inflation.

We now discuss some sensitivity exercises. First, not all stages of production are equally open to international trade. To investigate the importance of this heterogeneity, we infer the degree of openness by production stage for Canada using information in the World Input-Output Database (WIOD). We then re-estimate the optimal weights on the targeting variables in each monetary policy rule and the associated welfare loss. The results are reported in Appendix E. We find that the qualitative results are similar to our baseline case. This means that the observed heterogeneity in openness across production stages does not alter the basic results (at least for Canada).

Second, we explore different degrees of price stickiness across production stages. In particular, we consider two extreme cases: (i) sticky prices only in the downstream sector (but flexible prices in the upstream sector), or $\alpha_1 = 0$ and $\alpha_2 = 0.66$; and (ii) sticky prices only in the upstream sector, or $\alpha_1 = 0.66$ and $\alpha_2 = 0$. Details about this exercise can be found in Appendix J. Under the standard Taylor rule, greater openness leads to a smaller welfare loss in case (i), but a greater welfare loss in case (ii). The intuition is similar to that of Fig. 2: greater openness means a greater share of the total employment in the upstream sector. Thus, a given price distortion in the upstream stage is more damaging than in the downstream stage.

Third, we study how the elasticity of substitution at each stage of production matters for the welfare, and how it relates to the degree of openness. Intuitively, a greater elasticity of substitution tends to magnify the misallocation from price stickiness. If the elasticity of substitution differs in the two stages of production, a greater substitution in the upstream stage magnifies the overall welfare loss, and the effect becomes stronger as the economy becomes more open. Details can be found in Appendix J.

Fourth, we vary the share of intermediate goods, ϕ, and compute the optimal weights under Policy Rule 1. Fig. 3 traces out the estimated optimal relative weight on the upstream sector inflation as a function of the share of intermediate goods in production. The estimations show that, the optimal relative weight on the upstream sector inflation goes up as intermediate goods rise in importance.

Since CPI inflation in theory is a weighted average of the final-stage producer price inflation and the real exchange rate appreciation, we compute the implied weight on the CPI inflation in the optimal rule that targets stage-specific producer price inflation rates.[20] In Fig. 4, we trace out the ratio of the optimal coefficient on the upstream inflation index and the sum of the coefficient on the upstream inflation and the implied coefficient on CPI as a function of the share of intermediate goods. A clear upward trend suggests that, in the optimal simple rule, the weight on CPI should decline as the intermediate goods rise in relative importance. In other words, targeting CPI alone becomes increasingly sub-optimal as supply chains rise in importance.

4.4. Effects of a higher import tariff

Trade frictions can be thought of as a reduction in an economy's openness. Let us consider a case of doubling the import tariff (a change to $T = 2$). In such a scenario, the shares of the demand for domestic goods in the two production stages become $\bar{a}_1 = 0.73$ and $\bar{a}_2 = 0.74$, which are larger than those in the original calibration. The direction of the change is exactly as predicted by Proposition 1.

While the central bank cannot undo the increase in tariff directly, it can re-optimize by choosing a different set of coefficients on the variables in the monetary policy rule. We compute the new optimal weights

[20] As shown in the expression for CPI in Section 3.3, the weight on final stage estimated coefficient is set to be γ, while the weight on the estimated coefficient of exchange rate is set to be $1 - \gamma$.

Table 4
Optimal alternative simple rules of monetary policy with higher imported tariff.

	π_{1H}	π_{2H}	π_{PPI}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{i}_{t-1}$	Welfare loss
P1	5.0113	8.8786			0.2298	−0.7860	0.1629	1.182
P2				5.2378	0.0000		0.0007	2.144
P3			9.9988		0.1000		1.0509	1.223
P4			9.3569	0.0487	0.0766	−0.5579	0.1445	1.188
Peg								3.310

Notes: PPI index (sales-weighted): $\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$ with $\omega = \frac{P_{1h}(Y_{1h}+Y^X_{1h})}{P_{1h}(Y_{1h}+Y^X_{1h})+P_{2h}(Y_{2h}+Y^X_{2h})}$.
CPI index: $\pi_{CPI,\,t} = \pi_t$.

Table 5
Optimal alternative simple rules of monetary policy with lower price stickiness in upstream production.

	π_{1H}	π_{2H}	π_{PPI}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{i}_{t-1}$	Welfare loss
P1	3.1846	9.8760			0.0100	−0.5776	0.0328	1
P2				5.0515	0.0001		0.0038	1.889
P3			9.9981		0.1001		1.0715	1.083
P4			9.8058	0.0240	0.0110	−0.6126	0.0174	1.038
Peg								3.101

Notes: PPI index (sales-weighted): $\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$ with $\omega = \frac{P_{1h}(Y_{1h}+Y^X_{1h})}{P_{1h}(Y_{1h}+Y^X_{1h})+P_{2h}(Y_{2h}+Y^X_{2h})}$.
CPI index: $\pi_{CPI,\,t} = \pi_t$.

and new welfare losses for the best simple rule, the classic Taylor rule, the Galí-Monacelli rule, a rule that includes both PPI and CPI inflation as well as the output gap and real exchange rate but not stage-specific producer price inflation, and finally an exchange rate peg.

The results are reported in Table 4. Note that the welfare losses are relative to the case of Policy Rule 1 before the tariff increase (i.e., relative to the welfare for Policy Rule 1 in Table 2). Comparing across different policy rules, it is still the case that Policy Rule 1 that includes stage-specific producer price inflation is the best monetary policy rule. Afterwards, including PPI inflation and the real exchange rate would beat the classic Taylor rule. The exchange rate peg and the classic Taylor yield the biggest and the second biggest welfare losses, respectively.

Recall from Proposition 1 that, a higher import tariff would reduce the optimal weight on the upstream sector inflation in the monetary policy rule relative to that on the final stage inflation. This can be confirmed in our numerical exercises. The ratio of the optimal relative weight on the upstream producer price inflation under Policy Rule 1 has changed from 0.647 in the case of $T = 1$ in Table 2 to 0.564 in the case of $T = 2$ in Table 4.[21]

It is important to note that a higher tariff reduces welfare directly, as we can see from the greater welfare losses in Table 4 relative to their counterparts in Table 2, in spite of the best adjustments made by the central bank. If the central bank does not re-optimize, the welfare loss would have been even greater.

Since Policy Rule 1 already includes the real exchange rate, it implies that the central bank cannot offset the effects of a higher import tariff by simply changing the exchange rate. An appreciation in the domestic currency reduces the cost of imported intermediate inputs or imported final consumption goods, but also increases the prices of both domestically produced intermediate goods and final goods. Since the foreign demand is price-elastic, firms will experience a reduction in revenue from exporting.

4.5. Asymmetric price stickiness

We now consider uneven price stickiness in different stages of production. Cornille and Dossche (2008) and Nakamura and Steinsson (2008) argue that the duration of price contracts in the upstream production stages is shorter than the downstream stages. For instance, Nakamura and Steinsson (2008) document that the median price contract for finished producer goods in 1998–2005 lasts for 8.7 months, while the median duration of price contracts for intermediate goods is about 7.0 months.

To investigate the implications of such difference, we reduce the Calvo pricing parameter in the first stage of production to be $\alpha_1 = 0.5$, indicating an average length of price contracts of 2 quarters.

The estimated results are shown in Table 5, where the welfare loss of Policy Rule 1 in the table has been normalized to be one. The loss becomes smaller as compared to the baseline case in Table 2 since the prices are less sticky overall. Furthermore, the optimal relative weight on the upstream producer price inflation also becomes smaller.[22] Intuitively, it is beneficial to put more weight on those prices that are reset less frequently, i.e., downstream prices in this case, because resource misallocation is otherwise more severe.

4.6. Additional loss from sticky monetary policy rules

If a central bank adopted a policy rule that was optimal when the economy had a lower degree of participation in the global value chains, but did not update the rule as the participation has increased, what would the additional welfare cost be?

To investigate this, we continue with Canada as the baseline economy, with the same exogenous shocks as specified in Table 1. We choose $\gamma = 0.67$ (to match the 33% import share in GDP in the data) and $\bar{a}_1 = \bar{a}_2 = 0.69$ (to match the 31% export share in the data) in 2017. Similarly, we choose $\gamma = 0.75$ and $\bar{a}_1 = \bar{a}_2 = 0.74$ in 1987. The data suggests that the country has become more involved in the global trade from 1987 to 2017 as both the import and export shares have grown.

By computing an optimal CPI-based Taylor rule for the period around 30 years ago, we can then estimate the welfare loss if the central bank had used the old CPI-based Taylor rule in today's world. Table 6 shows the estimated welfare loss in 2017 if the central bank had

[21] Note that 0.647 comes from 6.3861/9.8675 in Table 2 while 0.564 comes from 5.0113/8.8786 in Table 4.

[22] This is consistent with the findings in Gong et al. (2016). They argue that, when the degree of price stickiness for intermediate-goods production is high, the central bank should follow intermediate-goods PPI-based rule. However, in their paper, there is no labor allocation distortion between production stages since labor is assumed to be used only in the production of intermediate goods.

Table 6
Welfare loss for adopting old policy rules estimated in 1987.

	π_{1H}	π_{2H}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{i}_{t-1}$	Welfare loss
New Rule	7.4579	9.5552		0.0187	−0.4054	0.1636	1
Taylor Rule			1.5	0.5			5.8715
Old Rule 1			5.6626	0.0001		0.0043	1.5217
Old Rule 2	7.0811	9.8512		1.3082	−1.8679	0.1636	1.1912

Notes: Old policy rules are estimated from Canada data in 1987, while the new policy rule is estimated from 2017.
CPI index: $\pi_{CPI,t} = \pi_t$.

continued to use an old optimal rule estimated in 1987. The welfare loss for the best new rule is normalized to be one.

The “Old Rule 1” and “Old Rule 2” refer to the optimal CPI-based Taylor rule and the optimal stage-specific PPI-based Taylor rule in 1987, respectively. From the table, we can see that the old Taylor rule (estimated optimally for 1987) generates a welfare loss in 2017. Furthermore, by comparing the two cases under “Old Rule”, if the country used to implement an optimal stage-specific PPI-based policy rule and applies to today, the welfare loss is smaller compared with the case of adopting an old CPI-based policy rule.

It is worth noting that, in estimating the old optimal CPI-based Taylor rule for 1987, we already assume two production stages. If, instead, there was a single production stage in 1987, and the world has evolved to be two production stages in 2017, then the welfare loss associated with using the old 1987 monetary policy in 2017 would be substantially bigger.

5. Concluding remarks

Supply chains are everywhere and are often global. This paper studies the implications of global supply chains on the design of optimal monetary policy using a small-open economy New Keynesian model with multiple stages of production. The optimal simple policy rule that produces the least welfare loss includes targeting separate producer price inflation in each production stage (in addition to output gap and real exchange rate).

Importantly, the optimal weights on the upstream sector inflation versus the final stage inflation are not proportional to the sectors' sales or value added. As an economy becomes more open, measured by the share of export in sales, the optimal relative weight on the upstream sector inflation will also rise. Separately, as intermediate goods become more important in the production, the optimal relative weight on the upstream sector inflation also rises. In both cases, the classic Taylor rule that targets only CPI inflation would become progressively more inferior (in the sense of an ever greater welfare loss relative to an optimal rule). As the production chain becomes longer, the optimal weights in the policy rule on the upstream sector inflation or the PPI inflation also increases.

Trade frictions can be thought of as a shock to an economy's openness. With a higher tariff, the optimal weights on various terms in the monetary policy rule would have to change. Importantly, a higher tariff reduces the welfare directly even if the central bank re-optimizes. In particular, the negative effect of a higher tariff cannot be offset completely by a change in the real exchange rate.

If we only consider aggregate price indices in the simple monetary policy rule, then targeting aggregate PPI inflation (as well as the output gap) is superior to just targeting CPI inflation in terms of a smaller welfare loss. Adding real exchange rate is even better. Still, no simple rule produces smaller welfare loss than the one that includes separate producer price inflation in each production stage on top of the output gap, the real exchange rate, and the lagged interest rate.

Is it feasible in practice to obtain separate producer price inflation for different production stages? Yes, as official statistical agencies in the United States and Australia already collect such data. For example, the US Bureau of Labor Statistics has a system of producer price indices featuring a four-stage vertical production chain (called the PPI Final Demand-Intermediate Demand indices).

Ironically, central banks use information on PPI inflation only to the extent that it helps to forecast CPI inflation. When PPI and CPI diverge, as they often do in recent periods, central banks would ignore PPI. However, our theory suggests that a monetary rule that produces an even smaller welfare loss includes producer price inflation directly, and doing so becomes more important precisely when the PPI and CPI inflation rates diverge.

The research in this paper can be extended in a number of directions. First, the model adopted in our analysis assumes producer currency pricing (PCP). It may be worth exploring how results may be modified as local currency pricing or dominant dollar pricing are assumed instead. Second, one may explore a broader set of exogenous shocks than in the current paper, including shocks on foreign input prices or foreign demand along the production chain.

Appendix A. Equilibrium characterization with *N*-stage of production in a small-open economy

A.1. The steady-state equilibrium

We first characterize the steady state with perfect foresight. The steady state is defined as the equilibrium under non-stochastic and constant exogenous variables. Since the whole economy does not change with timing, we can ignore the timing index t in all variables, and $A_n = 1$ for $n = 1, 2, \ldots, N$.

The optimal pricing decision for firms at stage n, $n = 1, 2, \ldots, N$, becomes

$$P_{nH}^{o} = \Psi_n = P_{nH} = \overline{P}_{nH}$$

and for $n = 2, \ldots, N$, we have

$$\overline{P}_n = \overline{P}_{(n-1)H}^{\gamma} \overline{P}_{(n-1)F}^{1-\gamma}$$

where $\overline{P}_{(n-1)F} = T\mathscr{E}P_{(n-1)F}^{*}$.

We solve for the price indices in terms of wages and derive the labor demand function. Note that $\Psi_n = \overline{P}_n^{\phi} W^{1-\phi}$ for $n = 2, 3, \ldots, N$ with $\Psi_1 = W$. By substituting $\overline{P}_n$, the relationship of output price index across adjacent stages is given by

$$P_{nH} = W^{1-\phi}\left(\overline{P}_n\right)^{\phi}$$

$$= W^{1-\phi}P^{\gamma\phi}_{(n-1)H}P^{(1-\gamma)\phi}_{(n-1)F}$$

for $n = 2, \ldots, N$ and $P_{1H} = W$.

By writing all price indices in terms of wage and exogenous variables through forward induction, we get

$$P_{nH} = W^{(1-\phi)\frac{1-(\gamma\phi)^{n-1}}{1-\gamma\phi}+(\gamma\phi)^{n-1}}(T\mathscr{E})^{\phi(1-\gamma)\frac{1-[\phi(1-\gamma)]^{n-1}}{1-\phi(1-\gamma)}}\Pi_{i=1}^{n-1}\left(P^*_{iF}\right)^{[\phi(1-\gamma)]^{n-i}} \tag{15}$$

for $n = 2, \ldots, N$ with $P_{1H} = W$.

Since $P_{nH}(u) = P_{nH}$ for $u \in [0,1]$ in steady state, by Eq. (12), for $n = 1, \ldots, N$, we have

$$Y^d_{nH,t}(u) = \overline{Y}^d_{nH,t}$$

Together with goods market clearing condition $Y_{nH} = \overline{Y}^d_{nH}$, and factor market demand function (9) and (8), for $n = 2, \ldots, N$, we get

$$\overline{Y}^d_n = \phi\frac{\Psi_n}{\overline{P}_n}\left[\overline{Y}^d_{nH} + Y^X_{nH}\right]$$

$$L^d_n = (1-\phi)\frac{\Psi_n}{W}\left[\overline{Y}^d_{nH} + Y^X_{nH}\right]$$

$$\overline{Y}^d_{(n-1)H} = \frac{\gamma\overline{P}_n\overline{Y}^d_n}{\overline{P}_{(n-1)H}}$$

where $\overline{Y}^d_{NH} = \gamma C P^{-(1-\gamma)}_{nH}\overline{P}^{1-\gamma}_{NF}$, and $L^d_1 = \frac{\Psi_1}{W}\overline{Y}^d_{1H}$. By substituting the price index and unit cost function in each stage, for $n = 1, \ldots, N$, through backward induction, the factor demand functions for labor can be written in an implicit form as

$$L^d_n = f\left(W, C, \mathscr{E}, T, P^*_{1F}, \cdots, P^*_{NF}, P^*_{nH}, \cdots, P^*_{NH}, Y^*_{nH}, \cdots, Y^*_{NH}\right) \tag{16}$$

By summing up labor demand across all stages, the total labor demand function becomes

$$L^d = \sum_{n=1}^{N} L^d_n \tag{17}$$

Therefore, the three Eqs. (1), (6), and (17) fully characterize the steady-state real wage, consumption and employment.

A.2. The flexible-price equilibrium

In this subsection, we solve for the flexible-price equilibrium similarly as for the steady-state equilibrium. In the flexible-price equilibrium, $\alpha_n = 0$ for $\forall n$. The optimal pricing decision for firms at stage n, $n = 1, 2, \ldots, N$, thus becomes

$$P^o_{nH,t} = \Psi_{n,t} = P_{nH,t} = \overline{P}_{nH,t}$$

and for $n = 2, \ldots, N$, we have

$$\overline{P}_{n,t} = \overline{P}^{\gamma}_{(n-1)H,t}\overline{P}^{1-\gamma}_{(n-1)F,t}$$

Similar to the steady-state case, we solve for the price indices in terms of wages and productivity. Note that $\Psi_{n,t} = \overline{P}^{\phi}_{n,t}W^{1-\phi}_t/A_{n,t}$ for $n = 2, 3, \ldots, N$ with $\Psi_1 = W_t/A_{1,\,t}$. By substituting $\overline{P}_{n,t}$, the relationship of price index across adjacent stages is given by

$$P_{nH,t} = W^{1-\phi}P^{\gamma\phi}_{(n-1)H,t}P^{(1-\gamma)\phi}_{(n-1)F,t}$$

for $n = 2, \ldots, N$ and $P_{1H,\,t} = W_t/A_{1,\,t}$.

By writing all price indices in terms of wage through forward induction, we similarly get

$$P_{nH,t} = W^{(1-\phi)\frac{1-(\gamma\phi)^{n-1}}{1-\gamma\phi}+(\gamma\phi)^{n-1}}\left(\mathscr{E}_tT_t\right)^{\phi(1-\gamma)\frac{1-[\phi(1-\gamma)]^{n-1}}{1-\phi(1-\gamma)}}$$

$$\cdot\Pi_{i=1}^{n-1}\left(P^*_{iF}\right)^{[\phi(1-\gamma)]^{n-i}}\cdot\Pi_{i=1}^{n}\left(A_{i,t}\right)^{-(\gamma\phi)^{n-i}}$$

for $n = 2, \ldots, N$ with $P_{1H,\,t} = W_t/A_{1,\,t}$.

Due to flexible prices, the expressions for factor market in each stage of production are exactly the same as in the steady-state case. Therefore, we can derive labor demand function in each stage, i.e., $n = 1, \dots, N$, as

$$L^{fd}_{n,t} = f\left(W_t, C_t, \mathscr{E}_t, T_t, P^*_{1F,t}, \cdots, P^*_{NF,t}, P^*_{nH,t}, \cdots, P^*_{NH,t}, Y^*_{nH,t}, \cdots, Y^*_{NH,t}, A_{1,t}, \cdots, A_{N,t}\right)$$

where we denote the labor demand under flexible prices with an upper symbol f.

The total labor demand function becomes

$$L^{fd}_t = \sum_{n=1}^{N} L^{fd}_{n,t} \tag{18}$$

The three Eqs. (1), (6), and (18) fully characterize the real wage, consumption and employment in the flexible-price equilibrium, where the consumption can be written in

$$C^f_t = f\left(T_t, P^*_{1F,t}, \cdots, P^*_{NF,t}, P^*_{nH,t}, \cdots, P^*_{NH,t}, Y^*_{nH,t}, \cdots, Y^*_{NH,t}, A_{1,t}, \cdots, A_{N,t}\right)$$

which can be re-written in log-linearized form, i.e.,

$$c^f_t = f\left(t_t, p^*_{1F,t}, \cdots, p^*_{NF,t}, p^*_{nH,t}, \cdots, p^*_{NH,t}, y^*_{nH,t}, \cdots, y^*_{NH,t}, a_{1,t}, \cdots, a_{N,t}\right)$$

By Euler Eq. (4), the IS curve is characterized by

$$c^f_t = E_t\left(c^f_{t+1}\right) - \frac{1}{\sigma}[i_t - E_t(\pi_{t+1}) - \rho]$$

which implies the natural rate of interest as

$$\bar{r}r_t = \rho + \sigma E_t\left\{c^f_{t+1} - c^f_t\right\} \tag{19}$$

A.3. The sticky-price equilibrium

We now derive New Keynesian Phillips curves for each stage as a function of the relative price gap and the output gap, and characterize the equilibrium with sticky prices. Similar to the derivation in Galí (2015), in each stage of production $n = 1, 2, \dots, N$, firms' optimal pricing decision gives

$$\pi_{n,t} = \beta E_t \pi_{n,t+1} + \lambda_n \hat{\gamma}_{n,t}$$

where $\lambda_n = \dfrac{(1-\beta\alpha_n)(1-\alpha_n)}{\alpha_n}$ and $\hat{\gamma}_n$ is the log-deviation of real marginal cost from steady-state equilibrium, i.e.,

$$\hat{\gamma}_{n,t} = \ln(\Psi_{n,t}/P_{nH,t}) - \ln(\Psi_n/P_{nH})$$

where Ψ_n and P_{nH} are the marginal cost and aggregate price in stage n, respectively, in the steady-state equilibrium.

Given $P_{nH,t} = \overline{P}_{nH,t}$ for $n = 1, 2, \dots, N$ and the production cost function, we have for stages $n = 2, \dots, N$

$$\hat{\gamma}_{n,t} = \gamma\phi\hat{g}_{nH,t} + (1-\gamma)\phi\hat{g}_{nF,t} + (1-\phi)(\hat{w}_t - \hat{p}_{nH,t}) - a_{n,t}$$

where $\hat{g}_{nH,t}$ and $\hat{g}_{nF,t}$ are the log-deviation of the relative output price gap with respect to input prices from the steady-state equilibrium, i.e., $\hat{g}_{nH,t} = ln\left(\dfrac{\overline{P}_{(n-1)H,t}}{P_{nH,t}}\right) - ln\left(\dfrac{\overline{P}_{(n-1)H}}{P_{nH}}\right)$, and $\hat{g}_{nF,t} = ln\left(\dfrac{\overline{P}_{(n-1)F,t}}{P_{nH,t}}\right) - ln\left(\dfrac{\overline{P}_{(n-1)F}}{P_{nH,t}}\right)$. Since $\overline{P}_{(n-1)H,t} = P_{(n-1)H,t}$, $\hat{g}_{nH,t}$ also indicates the log-deviation of the relative output price gap between adjacent stages n and $n-1$ from the steady-state equilibrium. By the definitions of $\hat{g}_{nH,t}$ and $\hat{g}_{nF,t}$, we also have $\hat{p}_{nH,t} = \hat{p}_{NH,t} + \Sigma^N_{i=n+1}\hat{g}_{iH,t}$ for $n = 1, \dots, N-1$.

Following Huang and Liu (2005), without loss of generality, we assume $\psi = 0$. Then, from Eq. (3), we have $w_t - p_t = \sigma c_t$. By substituting $\hat{w}$ and $\hat{p}_{nH,t}$ into the real marginal cost function, for $n = 2, \dots, N-1$, we obtain

$$\begin{aligned}\hat{\gamma}_{n,t} = {} & \gamma\phi\hat{g}_{nH,t} + (1-\gamma)\phi\hat{g}_{nF,t} \\ & + (1-\phi)\left[\sigma\hat{c}_t - (1-\gamma)\hat{p}_{NH,t} + (1-\gamma)\hat{p}_{NF,t} - \Sigma^N_{i=n+1}\hat{g}_{iH,t}\right] - a_{n,t}\end{aligned}$$

and for $n = 1$ or N

$$\hat{\gamma}_{N,t} = \gamma\phi\hat{g}_{NH,t} + (1-\gamma)\phi\hat{g}_{NF,t} + (1-\phi)\left[\sigma\hat{c}_t - (1-\gamma)\hat{p}_{NH,t} + (1-\gamma)\hat{p}_{NF,t}\right] - a_{N,t}$$

$$\hat{\gamma}_{1,t} = \sigma\hat{c}_t - (1-\gamma)\hat{p}_{NH,t} + (1-\gamma)\hat{p}_{NF,t} - \Sigma^N_{i=2}\hat{g}_{iH,t} - a_{1,t}$$

Note that $\hat{p}_{nF,t} = \hat{e}_t$ and $\hat{g}_{nF,t} = \hat{e}_t - \hat{p}_{nH,t}$ for $n = 1, \ldots, N$, and $\hat{q}_t = \gamma(\hat{e}_t - \hat{p}_{NH,t})$. We can simplify the above system of equations by plugging in $\hat{p}_{nF,t}$, $\hat{g}_{nF,t}$, and replace $\hat{e}_t$ with the real exchange rate $\hat{q}_t$.

After log-linearizing the Euler equation of the household around the steady state and subtracting the natural rate IS curve, we obtain the IS curve with sticky prices as

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}\left[\hat{i}_t - E_t(\pi_{t+1})\right]$$

The law of motion for the relative price gap between stages n and $n-1$, for $n = 2, 3, \ldots, N$, is characterized by

$$\hat{g}_{nH,t} = \hat{g}_{nH,t-1} + \pi_{(n-1)H,t} - \pi_{nH,t}$$

Given the policy rule $\{\hat{i}_t, \hat{e}_t\}$, the risk-sharing condition (7), the IS curve, the stage-specific Phillips curves, and the law of motion for the relative price gap fully pin down the dynamic equilibrium under sticky prices.

A.4. Stage-specific employment in a small-open economy with N-stage production

We derive the stage-specific employment gap in terms of output gap and relative price gap. By the factor demand function (8), (9), and (12) in each stage, and substituting with the unit cost, for $n = 2, 3, \ldots, N$, we have

$$lnL_{n,t} = ln(1-\phi) + \phi\left[ln\overline{P}_{n,t} - lnW_t\right] - lnA_{n,t} + ln\left[Y^d_{nH,t} + Y^X_{nH,t}\right] + d_{n,t} \tag{20}$$

where $d_{n,t} = ln\left(\int_0^1 \left(\frac{P_{n,t}(u)}{P_{n,t}}\right)^{-\theta} du\right)$ and $lnL_{1,t} = -lnA_{1,t} + ln\left[Y^d_{1H,t} + Y^X_{1H,t}\right] + d_{1,t}$.

By the factor demand function for intermediate goods and labor in each stage, i.e., Eqs. (8) and (9), for $n = 2, 3, \ldots, N$, we obtain

$$lnL_{n,t} = ln\left(\frac{1-\phi}{\phi}\right) + ln\overline{P}_{n-1,t} - lnW_t + ln\overline{Y}^d_{n,t}$$

Also, note that

$$Y^d_{(n-1)H,t} = \frac{\gamma \overline{Y}^d_{n,t} \overline{P}_{n,t}}{\overline{P}_{(n-1)H,t}}$$

Then, by substituting $lnY^d_{nH,t}$ using $lnL_{n,t}$ and $ln\overline{Y}^d_{(n+1),t}$ into Eq. (20), we obtain via backward induction the relationship for the stage-specific employment, i.e., for $n = 2, 3, \ldots, N$,

$$l_{n,t} = l_{n+1,t} + d_{n,t} + F_n\left(\hat{c}_t, e_t, t_t, a_{1,t}, \ldots, a_{N,t}, p^*_{1F,t}, \ldots, p^*_{NF,t}, p^*_{nH,t}, \ldots, p^*_{NH,t}, y^*_{nH,t}, \ldots, y^*_{NH,t}, \hat{g}_{1,t}, \ldots, \hat{g}_{N,t}\right)$$

where $l_{n,t} = lnL_{n,t}$.

Appendix B. The aggregate CPI inflation with two-stage production

Given exogenous foreign variables and import tariff to be constant, the aggregate CPI inflation index can be written as

$$\pi_t = \gamma\pi_{2H,t} + (1-\gamma)\Delta e_t$$

Since $\hat{q}_t = \gamma(\hat{e}_t - \hat{p}_{2H,t})$, we have

$$\Delta e_t = \frac{\Delta\hat{q}_t}{\gamma} + \pi_{2H,t}$$

Then, the aggregate CPI inflation can be re-written as

$$\begin{aligned}\pi_t &= \gamma\pi_{2H,t} + (1-\gamma)\Delta e_t \\ &= \pi_{2H,t} + \frac{1-\gamma}{\gamma}\Delta\hat{q}_t\end{aligned}$$

Appendix C. Stage-specific employment with two-stage production

We derive an explicit expression for the employment gap with two stages of production, i.e., $N = 2$. As specified in Section A.4, and also note that $\hat{w}_t = \gamma\hat{p}_{2H,t} + (1-\gamma)\hat{p}_{2f,t} + \sigma\hat{c}_t$, $\hat{p}_{1H,t} = \hat{g}_{2,t} + \hat{p}_{2H,t}$, and $\hat{p}_{1F,t} = \hat{p}_{2F,t} = \hat{e}_t$, we have for the second stage[23]

[23] We have imposed the assumption of $\psi = 0$.

$$\begin{aligned}\hat{l}_{2,t} &= \phi\left[\gamma\hat{p}_{1H,t}+(1-\gamma)\hat{p}_{1F,t}-\hat{w}_t\right]+\overline{a}_2\left[\gamma\hat{p}_{2H,t}+(1-\gamma)\hat{p}_{2F,t}-\hat{p}_{2H,t}+\hat{c}_t\right]\\ &\quad +(1-\overline{a}_2)(\hat{e}_t-\hat{p}_{2H})-a_{2,t}+d_{2,t}\\ &= \phi\left[\gamma\hat{g}_{2,t}-\sigma\hat{c}\right]+\overline{a}_2\left[(1-\gamma)(\hat{e}_t-\hat{p}_{2H,t})\right]+(1-\overline{a}_2)(\hat{e}_t-\hat{p}_{2H,t})-a_{2,t}+d_{2,t}\\ &= (\overline{a}_2-\phi\sigma)\hat{c}_t+\phi\gamma\hat{g}_{2,t}+\frac{1-\overline{a}_2\gamma}{\gamma}\hat{q}_t-a_{2,t}+d_{2,t}\end{aligned}$$

where, in the first equality, we have used $Y_{1H,t}^d=\frac{\gamma\overline{P}_{2,t}\overline{Y}_{2,t}^d}{\overline{P}_{1H,t}}$, $Y_{1H,t}^{Xd}=\frac{Y_{1H}^*P_{1H}^*\mathscr{E}_t}{P_{1H,t}}$, $Y_{2H,t}^d=\frac{\gamma C_tP_t}{\overline{P}_{2H,t}}$, and $Y_{2H,t}^d=\frac{Y_{2H}^*P_{2H}^*\mathscr{E}_t}{P_{2H,t}}$; the last equality uses the condition that $\hat{q}_t=\gamma(\hat{e}_t-\hat{p}_{2H,t})$.

For the first stage, the employment is given by

$$\begin{aligned}\hat{l}_{1,t} &= \overline{a}_1\left(\widehat{\overline{p}}_{2,t}+\widehat{\overline{y}}_{2,t}^d-\hat{p}_{1H,t}\right)+(1-\overline{a}_1)(\hat{e}_t-\hat{p}_{1H,t})+d_{1,t}-a_{1,t}\\ &= \overline{a}_1\left(\widehat{\overline{p}}_{2,t}+\hat{w}_t+\hat{l}_{2,t}-\widehat{\overline{p}}_{2,t}-\hat{p}_{1H,t}\right)+(1-\overline{a}_1)(\hat{e}_t-\hat{p}_{1H,t})+d_{1,t}-a_{1,t}\\ &= \overline{a}_1\hat{l}_{2,t}+\overline{a}_1\left[\gamma\hat{p}_{2H,t}+(1-\gamma)\hat{e}_t+\sigma\hat{c}_t-\hat{g}_{2H,t}-\hat{p}_{2H,t}\right]+(1-\overline{a}_1)\left[\hat{e}_t-\hat{p}_{2H,t}-\hat{g}_{2H,t}\right]+d_{1,t}-a_{1,t}\\ &= [-\overline{a}_1\sigma\phi+\overline{a}_1\overline{a}_2+\overline{a}_1\sigma]\hat{c}_t+[\overline{a}_1\phi\gamma-1]\hat{g}_{2H,t}\\ &\quad +\frac{1+\overline{a}_1-\overline{a}_1\gamma-\overline{a}_1\overline{a}_2\gamma}{\gamma}\hat{q}_t+\overline{a}_1d_{2,t}+d_{1,t}-a_{1,t}-\overline{a}_1a_{2,t}\end{aligned}$$

where the second equality uses the condition that $\hat{l}_{2,t}=\widehat{\overline{y}}_{2,t}^d-\hat{w}_t+\widehat{\overline{p}}_{2,t}$.

Appendix D. The steady-state share of the domestic demand in total demand with respect to import tariff

We characterize how the import tariff affects the steady-state share of the domestic demand in total demand in both production stages, i.e., $\overline{a}_1$ and $\overline{a}_2$, as specified in Section 3.

By the definition of $\overline{a}_2$, we have

$$\begin{aligned}\frac{\overline{a}_2}{1-\overline{a}_2} &= \frac{\gamma CP}{Y_{2H}^*P_{2H}^*\mathscr{E}}\\ &= \frac{\gamma C^*(P_t^*)^{1/\sigma}}{Y_{2H}^*P_{2H}^*}P^{1-\frac{1}{\sigma}}\mathscr{E}^{1/\sigma}\\ &= \frac{\gamma C^*(P_t^*)^{1/\sigma}}{Y_{2H}^*P_{2H}^*}\left\{\left[W^{1-\phi+\gamma\phi}(\mathscr{E}T)^{(1-\gamma)\phi}(P_{1F}^*)^{(1-\gamma)\phi}\right]^{\gamma}\left[\mathscr{E}TP_{2F}^*\right]^{1-\gamma}\right\}^{1-1/\sigma}\mathscr{E}^{1/\sigma}\end{aligned}$$

where the second equality uses $C=C^*\left(\frac{\mathscr{E}P_t^*}{P_t}\right)^{1/\sigma}$ and the third equality uses the condition specified in Section 3.1. Since $W=\mathscr{E}P^*(C^*)^{\sigma}$ under the assumption of $\psi=0$, by plugging W into the expression of $\overline{a}_2/(1-\overline{a}_2)$, the domestic price indices all cancel out (including the nominal exchange rate) and thus we have

$$\frac{\overline{a}_2}{1-\overline{a}_2}=f_2(*)\cdot T^{(1-\gamma)(1+\phi\gamma)\left(1-\frac{1}{\sigma}\right)}$$

where $f_2(*)$ is a function of exogenous foreign variables.

For the first stage, we have

$$\begin{aligned}\frac{\overline{a}_1}{1-\overline{a}_1} &= \frac{\gamma\overline{P}_2\overline{Y}_2^d}{\mathscr{E}P_{1H}^*Y_{1H}^*}\\ &= \frac{\gamma\phi P_{2H}\left(Y_{2H}^d+Y_{2H}^{Xd}\right)}{\mathscr{E}P_{1H}^*Y_{1H}^*}\\ &= \frac{\gamma\phi}{P_{1H}^*Y_{1H}^*}\frac{P_{2H}Y_{2H}^d}{\mathscr{E}\overline{a}_2}\\ &= \frac{\gamma\phi}{P_{1H}^*Y_{1H}^*\mathscr{E}}\frac{\gamma CP}{\overline{a}_2}\\ &= \frac{\gamma\phi}{P_{1H}^*Y_{1H}^*}\frac{Y_{2H}^*P_{2H}^*}{1-\overline{a}_2}\end{aligned}$$

where we have used $\frac{\overline{a}_2}{1-\overline{a}_2}=\frac{\gamma CP}{Y_{2H}^*P_{2H}^*\mathscr{E}}$ in the last equality. Therefore, for $\overline{a}_1$, we have

$$\frac{\bar{a}_1}{1-\bar{a}_1} = f_2(*) \cdot \frac{1}{1-\bar{a}_2}$$

where $f_1(*)$ are functions of exogenous foreign variables.

Appendix E. Heterogeneity in openness across stages

To explore the role of heterogeneity in openness across stages, we calibrate the export share in each stage for Canada by exploiting the input-output data from World Input-Output Database (WIOD). We choose the year of 2007 as the calibration target to avoid possible contamination from the Great Recession and the European sovereign debt crisis. The shares of exports along the production chain are set to be $\bar{a}_1 = 0.74$ and $\bar{a}_2 = 0.88$; other parameters are as specified in Table 1.[24] As shown in Policy Rule 1 of Table 7, the relative weight on the stage-specific PPI for upstream production is 0.552 (=5.4553/9.8780).

Table 7
Optimal alternative simple rules of monetary policy with heterogeneity in export share along production chain.

	π_{1H}	π_{2H}	π_{PPI}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{i}_{t-1}$	Welfare loss
P1	5.4553	9.8780			0.1189	−0.6283	0.1577	1
P2				5.1281	0.0001		0.0051	1.806
P3			9.9997		0.1000		1.0690	1.033
P4			9.9647	0.0022	0.0024	−0.4452	0.1478	1.005

Notes: PPI index (sales-weighted): $\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$ with $\omega = \frac{P_{1h}(Y_{1h} + Y^X_{1h})}{P_{1h}(Y_{1h} + Y^X_{1h}) + P_{2h}(Y_{2h} + Y^X_{2h})}$.

CPI index: $\pi_{CPI,t} = \pi_t$.

Appendix F. *N*-stage production in a closed economy

We consider the case of N-stage production in a closed-economy, and focus on the effects of lengthening of production chain on welfare loss function. We can similarly characterize the equilibrium (shown in Appendices F.3–F.5) as in the case of the open-economy model with two stages of production. In the closed-economy model, since the distortion from monopolistic competition is assumed to be corrected by a subsidy tax, the only distortion in the economy comes from sticky price. Thus, the flexible-price equilibrium is Pareto optimal and we can write each variable in the deviation from flexible-price equilibrium. The shocks considered in this section are stage-specific productivity shocks.

F.1. A utility-based objective welfare loss function for optimal monetary policy

Similarly to the derivation in Section 3.4, the household's utility function is given by

$$E\Sigma_{t=0}^{\infty}\beta^t[U(C_t) - V(L_t)]$$

where $U(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$ and $V(L_t) = \frac{L_t^{1+\psi}}{1+\psi}$.

A second-order Taylor expansion around steady state (C,L) for the period utility of consumption gives

$$U(C_t) - U = U_c C\left(\hat{c}_t + \frac{1-\sigma}{2}\hat{c}_t^2\right) + t.i.p.$$

where $\hat{c}_t$ denotes the log-deviation of consumption from steady state. To write the output gap in terms of the gap between the output with sticky-price and natural output (flexible-price equilibrium), the period utility of consumption can be re-written as

$$U(C_t) - U = U_c C\left(\tilde{c}_t + \frac{1-\sigma}{2}\tilde{c}_t^2 + (1-\sigma)c_t^f\tilde{c}_t\right) + t.i.p.$$

where $\tilde{c}_t = c_t - c_t^f$ and c_t^f is the log-deviation of consumption in the flexible-price equilibrium from the steady-state equilibrium.

By labor market clearing condition, we obtain the second-order Taylor expansion around steady state for the period utility of employment, i.e., $V(L_t)$, as

$$V(L_t) - V = V_L L\left\{\sum_{n=1}^{N}\frac{L_n}{L}\left[\hat{l}_{n,t} + \frac{1}{2}\hat{l}_{n,t}^2\right]\right\} + t.i.p.$$

where L_n/L is the share of labor demand in stage n in total labor demand under steady state, given by Eqs. (23) and (24) in Appendix F.3, and we have imposed the assumption of $\psi = 0$. More specifically, the stage-specific labor share under steady state is given by

$$\frac{L_n}{L} = (1-\phi)\phi^{N-n}, n = 2, 3, \ldots, N$$

[24] From the WIOD dataset, the export share in intermediate goods of Canada (including goods and services) is about 26% in 2007 and the export share in final demand is about 12%. It is worth noting that the export share in intermediate goods from WIOD is the ratio calculated through gross output of intermediate goods, which is lower than the corresponding ratio in value added term.

$$\frac{L_1}{L} = \phi^{N-1}$$

The period utility of employment can then be re-written as the gap between labor demand with sticky-price and the log-deviation of labor demand with flexible prices in each stage, i.e.,

$$V(L_t) - V = V_L L \left\{ \sum_{n=1}^{N} \frac{L_n}{L} \left[\tilde{l}_{n,t} + \frac{1}{2} \tilde{l}_{n,t}^2 + l_{n,t}^f \tilde{l}_{n,t} \right] \right\} + t.i.p \tag{21}$$

where $\tilde{l}_{n,t} = l_{n,t} - l_{n,t}^f$.

As shown in Appendix F.8, the stage-specific employment gap in terms of output gap and relative price gap for $n = 2, 3, \ldots, N-1$ are given by

$$\tilde{l}_{n,t} = \phi \left[\sum_{i=n}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t \right] + \tilde{l}_{n+1,t} - \left[\sum_{i=n+1}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t \right] + d_{n,t}$$

with

$$\tilde{l}_{N,t} = \phi [\tilde{g}_{N,t} - \sigma \tilde{c}_t] + \tilde{c}_t + d_{N,t}$$

$$\tilde{l}_{1,t} = \tilde{l}_{2,t} - \left[\sum_{i=2}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t \right] + d_{1,t}$$

where $d_{n,t} = ln \left(\int_0^1 \left(\frac{P_{n,t}(u)}{P_{n,t}} \right)^{-\theta} du \right)$ measures the price dispersion in stage n. Details can be found in Appendix F.8.

For simplicity, we denote

$$\tilde{l}_{n,t} = f_n(\tilde{g}_{n,t}, \ldots, \tilde{g}_{N,t}) + k(n) \tilde{c}_t + \sum_{i=n}^{N} d_{i,t}$$

$$\tilde{l}_{1,t} = f_1(\tilde{g}_{1,t}, \ldots, \tilde{g}_{N,t}) + k(1) \tilde{c}_t + \sum_{i=1}^{N} d_{i,t}$$

where $k(n) = (N-n)(1-\phi)\sigma + 1 - \phi\sigma$ for $n = 2, 3, \ldots, N$, and $k(1) = (N-1)(1-\phi)\sigma + 1$.

Then, by summing up $U(C_t) - U$ and $V(L_t) - V$ and also noting that the efficiency of steady state implies $U_c C = V_L L$ in the closed-economy, the first order terms all cancel out, and only the second-order terms are left. The welfare loss function as a fraction of steady state consumption is thus given by

$$\begin{aligned} W &= E_0 \sum_{t=0}^{\infty} \beta^t \frac{U(C_t) - V(L_t) - (U - V)}{U_c C} \\ &= -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -(1-\sigma) \tilde{c}_t^2 + \sum_{n=1}^{N} \frac{L_n}{L} [k(n) \tilde{c}_t + f_n(\tilde{g}_{n,t}, \ldots, \tilde{g}_{N,t})]^2 + \sum_{n=1}^{N} \theta \phi^{N-g} \lambda_n^{-1} \pi_{n,t}^2 \right\} \end{aligned} \tag{22}$$

where $\frac{L_n}{L} = (1-\phi)\phi^{N-n}$ for $n = 2, 3, \ldots, N$ and $\frac{L_1}{L} = \phi^{N-1}$.

Appendix G shows the welfare loss function for the case of $N = 2$ and $N = 3$ without abbreviation (i.e., expanding $\frac{L_n}{L}$, $k(n)$, and $f_n(\cdot)$ for n).

In general, monetary policy cannot attain a Pareto optimal allocation except in special cases with restrictions on productivity shocks. We proceed with the following proposition.

Proposition 2. In the closed-economy model with N stages of production and labor being used in each production stage (i.e., $0 < \phi < 1$), there is no monetary policy that can replicate flexible price equilibrium (Pareto-optimal allocation) unless the stage-specific productivity shocks satisfy $\sum_{i=1}^{n-1} \phi^{n-i-1} (\phi - 1) \Delta a_{i,\,t} + \Delta a_{n,\,t} = 0$ for $n = 2, \ldots, N$ and for all t.

F.2. Discussion about the terms and coefficients in welfare loss function

There are three main parts in the welfare loss function: (a) output gap, and terms measuring stage-specific unemployment gap written in output gap, (b) the relative price gap, and (c) terms measuring stage-specific inflation. More specifically, as showed in the expression of welfare loss function (22), the coefficients before output gap $\tilde{c}_t^2$ and the stage-specific inflation, i.e., $\pi_{n,\,t}^2$ for $\forall n$, are all positive.[25] Therefore, similar to the standard welfare loss function (e.g., Rotemberg and Woodford, 1999; Woodford, 2003), the objective for a benevolent central bank still includes stabilizing output gap and inflation (i.e., the final stage inflation corresponding to typical "inflation" in the literature).

[25] Details about the proof can be found in Appendix H.

Besides the output gap and final-stage inflation, there are many more terms included in the welfare loss function, classified by those measuring stage-specific unemployment gaps and stage-specific inflation. It suggests that the central bank should not only care about the output gap and CPI, but also need to pay attention to the variations in PPI inflation and the gaps of the real marginal cost in the production of intermediate goods.

Importantly, as shown in the expression of welfare loss function (22), by aggregating the terms of output gap $\tilde{c}_t^2$, the coefficient before output gap is $\sum_{n=1}^{N}\frac{L_n}{L}k(n)^2-(1-\sigma)$, which is a function of the production structure, and changes with the number of total production stage N. In contrast, the coefficient before CPI (i.e., the final stage inflation $\pi_{N,\,t}^2$) is a constant $\theta\lambda_N^{-1}$. That is to say, even the central bank follows the Taylor Rule, or the monetary rule suggested by Huang and Liu (2005), i.e., targeting both CPI and PPI, the optimal weights before output gap and CPI (or PPI) are changing with the production structure of the economy.

The welfare loss function with multi-stage production indicates that targeting both CPI and PPI are not satisfactory. Instead, the central bank needs to pay attention to all stage-specific inflation along production process, especially in the case of lengthening production chain. Those terms measuring stage-specific inflation, i.e., $\sum_{n=1}^{N}\theta\phi^{N-n}\lambda_n^{-1}\pi_{n,\,t}^2$, in welfare loss function have two important implications. On the one hand, given the total number of production stages N and the price stickiness being the same across different stages, the coefficients before inflation in downstream stages are larger compared with those in upstream stages. On the other hand, as the number of total stages N increases, there are more terms of upstream inflation included in the welfare loss function, while the terms for downstream inflation do not change. In the latter case, the relative importance of final stage inflation (i.e., CPI) in welfare loss function becomes smaller, while the inflation in upstream stages becomes relatively more important. That is to say, as the production length becomes longer, the central bank needs to care more about inflation in intermediate stages but less on the final stage inflation (i.e., CPI).

From the perspective of practice, if the stage-specific inflation cannot be attained, PPI, as a sales-weighted price index for intermediate goods across all stages, can be a proxy. But, in general, if PPI index is available, the information used to construct PPI index is likely enough to construct the stage-specific inflation. For instance, the PPI program of US Bureau of Labor Statistics not only constructs an aggregate PPI index, but also constructs stage-specific inflation indices in a four-stage vertical production framework with the same underlying data.[26] Their idea in constructing this system of indices is to choose the total number of stages and assign industries to stages of production in such a manner that simultaneously maximizes the forward goods flows along the vertical chain while minimizing backward flows and internal goods flows within the system.

F.3. The steady-state equilibrium

We first characterize the steady state with perfect foresight. We drop the time subscript t for all variables, and set $A_n = 1$ for $n = 1, 2, \ldots, N$. The optimal pricing decision for firms at stage n, $n = 1, 2, \ldots, N$, becomes

$$P_n^* = \Psi_n$$

By aggregate price expression (13), in the steady state, we have

$$P_n = P_n^* = \Psi_n$$

Now, we solve for the price indices in terms of the wages and derive the labor demand function. Note that $P_n = \overline{P}_{n+1}$ for $n = 1, 2, \ldots, N-1$, and $\Gamma_n = \overline{P}_n^{\phi} W^{1-\phi}$ for $n = 2, 3, \ldots, N$ with $\Gamma_1 = W$. By substituting $\overline{P}_n$, the relationship of price index between adjacent stages is given by

$$P_n = W^{1-\phi}(P_{n-1})^{\phi}$$

for $n = 2, \ldots, N$ and $P_1 = W$.

By rewriting all price indices in terms of wages, it comes

$$\begin{aligned} P_n &= W^{1-\phi^{n-1}}(P_1)^{\phi^{n-1}} \\ &= W \end{aligned}$$

for $n = 2, \ldots, N$ with $P_1 = W$.

Since $P_n(u) = P_n$ for $u \in [0, 1]$ in the steady state, we have

$$Y_{n-1,t}^d(u) = \overline{Y}_{n,t}^d$$

Together with the goods markets clearing condition $Y_{n,t} = \overline{Y}_{n+1,t}^d$, and factor market demand function (9) and (8), for $n = 2, \ldots, N-1$, we obtain

$$\overline{Y}_n^d = \phi\frac{\Gamma_n}{\overline{P}_n}\overline{Y}_{n+1}^d$$

$$L_n^d = (1-\phi)\frac{\Gamma_n}{W}\overline{Y}_{n+1}^d$$

where $Y_N = C$, $\overline{Y}_N = \phi\frac{\Gamma_N}{\overline{P}_N}C$, $L_N^d = (1-\phi)\frac{\Gamma_N}{W}C$, and $L_1^d = \frac{\Gamma_1}{W}\overline{Y}_2^d$. By substituting the price index and unit cost function in each stage, for $n = 2, \ldots, N$, the factor demand functions for both labor and composite intermediate goods are given by

[26] Details for the stage-specific inflation indices constructed by US Bureau of Labor Statistics can be found at https://www.bls.gov/ppi/fdidsummary.htm, or Weinhagen (2011).

$$\overline{Y}_n^d = \phi^{N+1-n} C$$

$$L_n^d = (1-\phi)\phi^{N-n} C \tag{23}$$

with $L_1^d = \overline{Y}_2^d$.[27]

By summing up the labor demand across all stages, the total labor demand function becomes

$$L^d = \sum_{n=2}^{N} \left[(1-\phi)\phi^{N-n} C \right] + \phi^{N-1} C \tag{24}$$

With the labor supply function (1) together with the price index, the labor supply in the steady state becomes

$$L^\psi C^\sigma = 1 \tag{25}$$

Given $L^d = L$, the two Eqs. (24) and (25) fully characterize the steady-state total consumption and total employment.

F.4. The flexible-price equilibrium

In order to obtain efficient allocation in the model economy, i.e., the natural rate of the output, we solve for the flexible-price equilibrium in a similar way as in the steady state. In the flexible-price equilibrium, $\alpha_n = 0$ for $\forall n$, and the optimal pricing decision for firms at stage n, $n = 1, 2, \ldots, N$, becomes

$$P_{n,t}^* = \Gamma_{n,t}$$

By the aggregate price expression (13), we have

$$P_{n,t} = P_{n,t}^* = \Gamma_{n,t}$$

Similar to the steady-state case, we solve for the price indices in terms of wages and productivity. Note that $P_{n,t} = \overline{P}_{n+1,t}$ for $n = 1, 2, \ldots, N-1$, and $\Gamma_{n,t} = \overline{P}_{n,t}^{\phi} W_t^{1-\phi} / A_{n,t}$ for $n = 2, 3, \ldots, N$ with $\Gamma_1 = W_t / A_{1,\,t}$. By substituting $\overline{P}_{n,t}$, the relationship of price index across adjacent stages is given by

$$P_{n,t} = W_t^{1-\phi} (P_{n-1,t})^{\phi} / A_{n,t}$$

for $n = 2, \ldots, N$ and $P_1 = W / A_{1,\,t}$.

By writing all price indices in terms of wage, we obtain

$$\begin{aligned} P_{n,t} &= W^{1-\phi^{n-1}} (P_1)^{\phi^{n-1}} \Pi_{g=2}^{n} A_{g,t}^{-\phi^{n-g}} \\ &= W \Pi_{g=1}^{n} A_{g,t}^{-\phi^{n-g}} \end{aligned} \tag{26}$$

for $n = 2, \ldots, N$.

Similar to the derivation for the steady-state case, the labor demand function in each stage in flexible-price equilibrium is given by

$$L_{n,t}^d = (1-\phi)\phi^{N-n} \Pi_{g=1}^{N} A_{g,t}^{-\phi^{N-g}} C_t \tag{27}$$

for $n = 2, 3, \ldots, N$ with $L_{1,t}^d = \frac{\phi}{1-\phi} L_{2,t}^d$. Details can be found in Appendix F.6.

Therefore, the total labor demand is given by

$$L_t^d = \eta \Pi_{n=1}^{N} A_{n,t}^{-\phi^{N-n}} C_t \tag{28}$$

where η is a constant, and it is given by

$$\eta = \sum_{n=2}^{N} \left[(1-\phi)\phi^{N-n} C \right] + \phi^{N-1}$$

By the labor supply function (1) together with the price index, we know

$$L^\psi C^\sigma = \Pi_{n=1}^{N} A_{n,t}^{\phi^{N-n}} \tag{29}$$

[27] The labor demand in each stage can be viewed as a form of backward deduction (which is helpful when taking log-linearization), i.e.,

$$\begin{aligned} L_n^d &= \phi L_{n+1}^d, \; n = 2, \ldots, N \\ L_1^d &= \frac{\phi}{1-\phi} L_2^d \end{aligned}$$

After taking log-deviation from the steady state for both the total labor demand (28) and the labor supply (29), we get

$$l_t^f = c_t^f - \left[\sum_{n=1}^{N} \phi^{N-n} a_{n,t}\right] \tag{30}$$

and

$$\psi l_t^f + \sigma c_t^f = \sum_{n=1}^{N} \phi^{N-n} a_{n,t} \tag{31}$$

where the variables in lower case with an upper symbol f denote the log-deviation of flexible price equilibrium from the steady state.

Therefore, the log-deviation of output from the steady state under flexible prices c_t^f is given by

$$c_t^f = \frac{1+\psi}{\psi+\sigma}\left[\sum_{n=1}^{N} \phi^{N-n} a_{n,t}\right]$$

By the Euler Eq. (4), the IS curve is characterized by

$$c_t^f = E_t\left(c_{t+1}^f\right) - \frac{1}{\sigma}\left[\hat{i}_t - E_t(\pi_{N,t+1})\right]$$

which yields the natural rate of interest as

$$\bar{r}r_t = \hat{i}_t - E_t(\pi_{N,t+1}) + \rho$$

$$= \rho + \sigma E_t\left\{c_{t+1}^f - c_t^f\right\}$$

Given the expression of output and the process of productivity shocks, we have

$$\bar{r}r_t = \rho + \frac{\sigma(1+\psi)}{\psi+\sigma} E_t\left[\sum_{n=1}^{N} \phi^{N-n} \Delta a_{n,t+1}\right] \tag{32}$$

where $\Delta a_{n,\,t} = a_{n,\,t} - a_{n,\,t-1}$ is the growth rate of productivity in stage n.

F.5. The sticky-price equilibrium

We now derive New Keynesian Phillips curves for each stage as a function of relative price gap and output gap, and characterize the equilibrium with sticky prices. Similar to the derivation in Galí (2015), in each stage of production $n = 1, 2, \ldots, N$, firms' optimal pricing decision gives

$$\pi_{n,t} = \beta E_t \pi_{n,t+1} + \lambda_n \tilde{\gamma}_{n,t}$$

where $\lambda_n = \frac{(1-\beta\alpha_n)(1-\alpha_n)}{\alpha_n}$ and $\tilde{\gamma}_n$ is the log-derivation of the real marginal cost from the flexible-price equilibrium, i.e.,

$$\tilde{\gamma}_{n,t} = ln(\Gamma_{n,t}/P_{n,t}) - ln\left(\Gamma_{n,t}^f/P_{n,t}^f\right)$$

where $\Gamma_{n,\,t}^*$ and $P_{n,\,t}^*$ are, respectively, the marginal cost and aggregate price in stage n in the flexible-price equilibrium.

Following Huang and Liu (2005), without a loss of generality, we assume that $\psi = 0$. Together with the labor supply function (29), for $n = 2, 3, \ldots, N-1$, the log-deviation of the real marginal cost can be written as a function of relative price gap and output gap, i.e.,

$$\tilde{\gamma}_{n,t} = \phi \tilde{q}_{n,t} + (1-\phi)\left[\sigma \tilde{c}_t - \sum_{i=n+1}^{N} \tilde{g}_{i,t}\right] \tag{33}$$

$$\tilde{\gamma}_{1,t} = \sigma \tilde{c}_t - \sum_{i=2}^{N} \tilde{g}_{i,t}$$

$$\tilde{\gamma}_{N,t} = \phi \tilde{g}_{N,t} + (1-\phi)\sigma \tilde{c}_t$$

where $\tilde{g}_{n,t}$ is the relative price gap between stage n and stage $n-1$, i.e., $\tilde{g}_{n,t} = ln\left(\frac{P_{n-1,t}}{P_{n,t}}\right) - ln\left(\frac{P_{n-1,t}^f}{P_{n,t}^f}\right)$. Details for Expression (33) can be found in Appendix F.7.

After log-linearizing the Euler equation around the steady state and subtracting the natural rate IS curve, we obtain the IS curve with sticky prices as

$$\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma}\left[i_t - E_t(\pi_{N,t+1}) - \bar{r}r_t\right]$$

where $\bar{r}r_t$ is the natural rate of interest.

The law of motion for the relative price gap between stage n and stage $n-1$, for $n=2, 3, \ldots, N$, is characterized by

$$\tilde{g}_{n,t} = \tilde{g}_{n,t-1} + \pi_{n-1,t} - \pi_{n,t} - \Delta g^f_{n,t}$$

where $\Delta g^f_{n,t} = g^f_{n,t} - g^f_{n,t-1}$. By Eq. (26), we have

$$\Delta g^f_{n,t} = \sum_{i=1}^{n-1} \phi^{n-i-1}(\phi-1)\Delta a_{i,t} + \Delta a_{n,t}$$

Give the monetary policy rule, the Phillips curve, IS curve, and the law of motion for the relative price gap fully pin down the dynamic equilibrium under sticky prices.

F.6. The labor demand function in the flexible-price equilibrium

Similar to the steady-state equilibrium, we derive the labor demand function in the flexible-price equilibrium. Note that, with flexible prices, $P_n(u) = P_n$ for $u \in [0, 1]$ and we then obtain

$$Y^d_{n-1,t}(u) = \overline{Y}^d_{n,t}$$

Together with goods markets clearing condition $Y_{n,t} = \overline{Y}^d_{n+1,t}$, and factor market demand function (9) and (8), for $n = 2, \ldots, N-1$, we obtain

$$\overline{Y}^d_{n,t} = \phi \frac{\Gamma_{n,t}}{\overline{P}_{n,t}} \overline{Y}^d_{n+1,t}$$

$$L^d_{n,t} = (1-\phi)\frac{\Gamma_{n,t}}{W_t} \overline{Y}^d_{n+1,t}$$

where $Y_{N,\,t} = C_t$, $\overline{Y}_{N,t} = \phi \frac{\Gamma_{N,t}}{\overline{P}_N} C_t$, $L^d_{N,t} = (1-\phi)\frac{\Gamma_{N,t}}{W_t} C_t$, and $L^d_{1,t} = \frac{\Gamma_{1,t}}{W_t} \overline{Y}^d_{2,t}$.

Note that $P_{n,t} = \overline{P}_{n+1,t}$ for $n = 1, 2, \ldots, N-1$, and $\Gamma_{n,t} = \overline{P}^{\phi}_{n,t} W^{1-\phi}_t / A_{n,t}$ for $n = 2, 3, \ldots, N$ with $\Gamma_1 = W_t/A_{1,\,t}$. By substituting the unit cost function in each stage, for $n = 2, \ldots, N$, we obtain the labor demand in each stage as follows:

$$\overline{Y}^d_{n,t} = \frac{\phi}{1-\phi} \frac{W_t}{P_{n-1,t}} L^d_{n,t}$$

and thus

$$L^d_{n,t} = \phi \frac{P_{n-1,t}}{P_{n,t}} \left(\frac{W_t}{P_{n-1,t}}\right)^{1-\phi} A^{-1}_{n,t} L^d_{n+1,t}$$

$$L^d_{1,t} = \frac{1}{A_{1,t}} \frac{\phi}{1-\phi} \frac{W_t}{P_{1,t}} L^d_{2,t}$$

One can derive the labor demand in each stage via backward induction (which is helpful when taking log-linearization), i.e.,

$$L^d_{n,t} = \phi L^d_{n+1,t}, n = 2, \ldots, N$$

$$L^d_{1,t} = \frac{\phi}{1-\phi} L^d_{2,t}$$

Note that $L^d_{N,t} = (1-\phi)\frac{\Gamma_{N,t}}{W_t} C_t$, which indicates

$$L^d_{N,t} = (1-\phi)\Pi^N_{g=1} A^{-\phi^{N-g}}_{g,t} C_t$$

Therefore, for $n = 2, 3, \ldots, N$, we obtain the labor demand function in each stage as

$$L^d_{n,t} = (1-\phi)\phi^{N-n}\Pi^N_{g=1} A^{-\phi^{N-g}}_{g,t} C_t$$

with $L^d_{1,t} = \frac{\phi}{1-\phi} L^d_{2,t}$.

F.7. The log-deviation of the real marginal cost from the flexible-price equilibrium

Note that, for $n = 2, 3, \ldots, N$, $\Gamma_{n,t} = \overline{P}^{\phi}_{n,t} W^{1-\phi}_t / A_{n,t}$ and $\overline{P}_{n,t} = P_{n-1,t}$. The log-deviation of the real marginal cost is given by

$$\tilde{\gamma}_{n,t} = ln(\Gamma_{n,t}/P_{n,t}) - ln\left(\Gamma^*_{n,t}/P^*_{n,t}\right)$$

$$= \phi\left[ln(P_{n-1,t}/P_{n,t}) - ln\left(P^*_{n-1,t}/P^*_{n,t}\right)\right] + (1-\phi)\left[ln(W_t/P_{n,t}) - ln\left(W^f_t/P^f_{n,t}\right)\right]$$

Denote $g_{n,\ t} = \ ln\ (P_{n-1,\ t}/P_{n,\ t})$ and $\tilde{g}_{n,t} = ln(P_{n-1,t}/P_{n,t}) - g^f_{n,t}$. For $n = 1, 2, \ldots, N-1$, we have

$$lnP_{n,t} = \sum_{i=n+1}^{N} g_{i,t} + lnP_{N,t}$$

$$\Leftrightarrow p_{n,t} = \sum_{i=n+1}^{N} g_{i,t} + p_{N,t}$$

Also, by the labor supply Eq. (29), by assuming $\psi = 0$, we have

$$w_t - p_{N,t} = \sigma c_t$$

Therefore, for $n = 2, 3, \ldots N-1$, the log-deviation of real marginal cost can be written as

$$\tilde{\gamma}_{n,t} = \phi \tilde{g}_{n,t} + (1-\phi)\left[\tilde{w}_t - \tilde{p}_{n,t}\right]$$

$$= \phi \tilde{g}_{n,t} + (1-\phi)\left[\sigma \tilde{c}_t + \tilde{p}_{N,t} - \tilde{p}_{n,t}\right]$$

$$= \phi \tilde{g}_{n,t} + (1-\phi)\left[\sigma \tilde{c}_t - \sum_{i=n+1}^{N} \tilde{g}_{i,t}\right]$$

with $\tilde{\gamma}_{N,t} = \phi \tilde{g}_{N,t} + (1-\phi)\sigma \tilde{c}_t$.

Similarly, for the first stage $n = 1$, since $\Gamma_1 = W_t/A_{1,\ t}$, we have

$$\tilde{\gamma}_{1,t} = \tilde{w}_t - \tilde{p}_{1,t}$$

$$= \sigma \tilde{c}_t - \sum_{i=2}^{N} \tilde{g}_{i,t}$$

F.8. Stage-specific employment gaps in a closed economy with N-stage production

We derive the stage-specific employment gap in terms of output gap and relative price gap. By the factor demand function (8), (9), and (12) in each stage, and substituting with the unit cost, for $n = 2, 3, \ldots, N$, we have

$$lnL_{n,t} = ln(1-\phi) + \phi\left[lnP_{n-1,t} - lnW_t\right] - lnA_{n,t} + ln\overline{Y}^d_{n+1,t} + d_{n,t}$$

$$\Leftrightarrow l_{n,t} = ln(1-\phi) + \phi\left[p_{n-1,t} - w_t\right] - a_{n,t} + ln\overline{Y}^d_{n+1,t} + d_{n,t}$$

where $d_{n,t} = ln\left(\int_0^1 \left(\frac{P_{n,t}(u)}{P_{n,t}}\right)^{-\theta} du\right)$ and $l_{1,t} = -a_{1,t} + ln\overline{Y}^d_{2,t} + d_{1,t}$.

By the factor demand function for intermediate goods and labor in each stage, i.e., Eqs. (8) and (9), for $n = 2, 3, \ldots, N$, we get

$$l_{n,t} = ln\left(\frac{1-\phi}{\phi}\right) + p_{n-1,t} - w_t + ln\overline{Y}^d_{n,t}$$

Note that $\overline{Y}^d_{N,t} = C_t$. Then, by substituting $ln\overline{Y}^d_{n+1,t}$, we obtain the relationship for the stage-specific employment, i.e., for the stage of $n = N$, via backward induction as

$$l_{N,t} = ln(1-\phi) + \phi\left[p_{N-1,t} - w_t\right] - a_{N,t} + c_t + d_{N,t}$$

for $n = 2, 3, \ldots, N-1$,

$$l_{n,t} = ln(\phi) + \phi\left[p_{n-1,t} - w_t\right] - a_{n,t} + l_{n+1,t} - \left[p_{n,t} - w_t\right] + d_{n,t}$$

for $n = 1$,

$$l_{1,t} = -a_{1,t} + l_{2,t} - \left[p_{1,t} - w_t\right] + d_{1,t}$$

As shown in Appendix F.7, for $n = 1, 2, \ldots, N$, we have $p_{n,\ t} = \sum_{i=n+1}^{N} g_{i,\ t} + p_{N,\ t}$, and, by assuming $\psi = 0$, $w_t - p_{N,\ t} = \sigma c_t$. The stage-specific employment can be written in terms of relative price and output as, for $n = N$,

$$l_{N,t} = ln(1-\phi) + \phi\left[g_{N,t} - \sigma c_t\right] - a_{N,t} + c_t + d_{N,t}$$

for $n = 2, 3, \ldots, N-1$,

$$l_{n,t} = ln(\phi) + \phi\left[\sum_{i=n}^{N} g_{i,t} - \sigma c_t\right] - a_{n,t} + l_{n+1,t} - \left[\sum_{i=n+1}^{N} g_{i,t} - \sigma c_t\right] + d_{n,t}$$

for $n = 1$,

$$l_{1,t} = -a_{1,t} + l_{2,t} - \left[\sum_{i=2}^{N} g_{i,t} - \sigma c_t\right] + d_{1,t}$$

By subtracting the corresponding equations for the flexible-price equilibrium, the stage-specific employment gap in terms of output gap and the relative price gap is given by, for $n = 2, 3, \ldots, N-1$,

$$\tilde{l}_{n,t} = \phi\left[\sum_{i=n}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t\right] + \tilde{l}_{n+1,t} - \left[\sum_{i=n+1}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t\right] + d_{n,t}$$

with

$$\tilde{l}_{N,t} = \phi[\tilde{g}_{N,t} - \sigma\tilde{c}_t] + \tilde{c}_t + d_{N,t}$$

$$\tilde{l}_{1,t} = \tilde{l}_{2,t} - \left[\sum_{i=2}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t\right] + d_{1,t}$$

Therefore, by forward induction, the stage-specific employment gap is given by, for $n = 2, 3, \ldots, N-1$,

$$\tilde{l}_{n,t} = \phi\left[\sum_{i=n}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t\right] + \tilde{l}_{n+1,t} - \left[\sum_{i=n+1}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t\right] + d_{n,t}$$

with

$$\tilde{l}_{N,t} = \phi[\tilde{g}_{N,t} - \sigma\tilde{c}_t] + \tilde{c}_t + d_{N,t}$$

$$\tilde{l}_{1,t} = \tilde{l}_{2,t} - \left[\sum_{i=2}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t\right] + d_{1,t}$$

Appendix G. The closed-form welfare loss function for the case of $N = 2$ and $N = 3$ in a closed economy

To illustrate the welfare loss function in the closed economy, we show the analytical welfare loss function for the cases of $N = 2$ and $N = 3$ without abbreviation. For the case of $N = 2$, by Appendix F.8, the stage-specific employment gap in terms of output gap and relative price gap is given by

$$\tilde{l}_{1,t} = (1 + \sigma - \sigma\phi)\tilde{c}_t + (\phi - 1)\tilde{g}_{2,t} + d_{1,t} + d_{2,t}$$

$$\tilde{l}_{2,t} = (1 - \sigma\phi)\tilde{c}_t + \phi\tilde{g}_{2,t} + d_{2,t}$$

Since $\frac{L_1}{L} = \phi$ and $\frac{L_2}{L} = 1 - \phi$, by plugging into Eq. (22), the welfare loss function with $N = 2$ is given by

$$W = -\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left\{\sigma\tilde{c}_t^2 + \phi(1-\phi)\left[\sigma\tilde{c}_t - \tilde{g}_{2,t}\right]^2 + \theta\lambda_2^{-1}\pi_{2,t}^2 + \theta\phi\lambda_1^{-1}\pi_{1,t}^2\right\}$$

which is exactly the same as in Huang and Liu (2005).

Similarly, for the case of $N = 3$, the stage-specific employment gap in terms of output gap and relative price gap is given by

$$\tilde{l}_{1,t} = (1 + 2\sigma - 2\sigma\phi)\tilde{c}_t + 2(\phi - 1)\tilde{g}_{3,t} + (\phi - 1)\tilde{g}_{2,t} + d_{1,t} + d_{2,t} + d_{3,t}$$

$$\tilde{l}_{2,t} = (1 + \sigma - 2\sigma\phi)\tilde{c}_t + (2\phi - 1)\tilde{g}_{3,t} + \phi\tilde{g}_{2,t} + d_{2,t} + d_{3,t}$$

$$\tilde{l}_{1,t} = (1 - \sigma\phi)\tilde{c}_t + \phi\tilde{g}_{3,t} + d_{3,t}$$

Since $\frac{L_1}{L} = \phi^2$, $\frac{L_2}{L} = \phi(1-\phi)$ and $\frac{L_3}{L} = 1 - \phi$, by plugging into Eq. (22), the welfare loss function with $N = 3$ is given by

$$W = -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t - (1-\sigma)\tilde{c}_t^2 + \phi^2\left[(1+2\sigma-2\sigma\phi)\tilde{c}_t + 2(\phi-1)\tilde{g}_{3,t} + (\phi-1)\tilde{g}_{2,t}\right]^2$$
$$+(1-\phi)\phi\left[(1+\sigma-2\sigma\phi)\tilde{c}_t + (2\phi-1)\tilde{g}_{3,t} + \phi\tilde{g}_{2,t}\right]^2$$
$$+(1-\phi)\left[(1-\sigma\phi)\tilde{c}_t + \phi\tilde{g}_{3,t}\right]^2$$
$$+\theta\lambda_3^{-1}\pi_{3,t}^2 + \theta\phi\lambda_2^{-1}\pi_{2,t}^2 + \theta\phi^2\lambda_3^{-1}\pi_{3,t}^2$$

Appendix H. The proof for a positive coefficient of the output gap in the welfare loss function in the closed economy

The coefficient of output gap $\tilde{c}_t^2$ in the welfare loss function (22) is given by

$$-(1-\sigma) + \sum_{n=1}^{N}\frac{L_n}{L}k(n)^2 \equiv f$$

Note that the stage-specific labor share under the efficient steady state yields

$$\frac{L_n}{L} = (1-\phi)\phi^{N-n}, n = 2, 3, \ldots, N$$

$$\frac{L_1}{L} = \phi^{N-1}$$

and $k(n) = (N-n)(1-\phi)\sigma + 1 - \phi\sigma$ for $n = 2, 3, \ldots, N$, and $k(1) = (N-1)(1-\phi)\sigma + 1$. If $\sigma > 1$, then obviously $f > 0$; otherwise, since $\phi < 1$ and $\sigma \le 1$, it is obvious that $k(n) \ge 1 - \sigma\phi > 0$ for $\forall n$.

Therefore, in this case,

$$f = -(1-\sigma) + \sum_{n=1}^{N}\frac{L_n}{L}k(n)^2$$

$$\ge -(1-\sigma) + \sum_{n=1}^{N}(1-\sigma\phi)\frac{L_n}{L}k(n)$$

$$= (1-\sigma\phi)-(1-\sigma)$$

$$= \sigma(1-\phi)>0$$

In other words, the coefficient on the output gap in the welfare loss function is always positive.

Appendix I. Trade balance and optimal simple monetary policy rules

Instead of imposing the risk-sharing condition as specified in Section 2, we now assume that the households have no access to the international asset market (i.e., they live in financial autarky). By construction, goods trade has to be balanced (and the risk-sharing condition no longer holds). Under this assumption, the aggregate expenditure must be equal to the aggregate income, i.e., $W_tL_t = P_tC_t$.

The balanced trade condition in the steady state also requires the value of exports to equal that of imports, i.e., $1 = \frac{\gamma}{\bar{a}_2}(1-\phi) + \phi\frac{\gamma^2}{\bar{a}_2\bar{a}_1}$. By replacing the risk-sharing condition with balanced trade, we estimate the general *nonlinear* model (with $N = 2$) and approximate the equilibrium by a second-order expansion. The shares of goods sold in the domestic markets in the two stages are set to be $\bar{a}_1 = \bar{a}_2 = 0.6$ in order to satisfy the balanced trade condition. All other parameters are the same as in Table 1.[28]

Table 8
Optimal alternative simple rules of monetary policy under trade balance.

	π_{1H}	π_{2H}	π_{PPI}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{i}_{t-1}$	Welfare loss
P1	3.0339	5.0303			2.1428	−3.4481	0.2182	1
P2				4.9154	0.0000		0.0001	1.752
P3			9.9707		0.1012		1.0215	1.055
P4			9.9997	0.0001	0.0000		0.7661	1.040
P5			6.0028	0.1431	2.0563	−3.3121	0.1726	1.011
P6	5.5358	9.9393			0.0021		0.5837	1.021
P7	5.5339				0.0002		0.7914	1.767
P8		2.9431			0.0012		0.7929	1.301
P9	5.5710	9.9870					0.5809	1.021
Peg								2.148

Notes: PPI index (sales-weighted): $\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$ with $\omega = \frac{P_{1h}(Y_{1h} + Y_{1h}^X)}{P_{1h}(Y_{1h} + Y_{1h}^X) + P_{2h}(Y_{2h} + Y_{2h}^X)}$.

CPI index: $\pi_{CPI,t} = \pi_t$.

[28] Under the assumption of a balanced trade, a shock on foreign consumption is inconsequential for the domestic economy.

With this new structure of the model, we re-estimate the optimal weights for each of the simple monetary policy rule discussed in the main text, and compute the associated welfare loss (relative to the best simple rule). Table 8 presents the result.

By construction, Policy Rule 1 that targets the producer price inflation in all stages of production, plus the output gap and the real exchange rate, is the best rule among the family of simple rules. Given the differences in the model structure, it is not surprising that the estimated optimal weights on various variables and the numerical values of the welfare losses for the policy rules are different from those in Section 4.2. However, it is noteworthy that the relative welfare ordering of the simple rules is the same as before. In particular, the conventional Taylor rule (Policy Rule 2) that targets only the CPI inflation and output gap is associated with a sizable additional welfare loss, even with optimally estimated weights on the targeting variables, when compared with the best simple rule. An exchange rate peg (Policy Rule 10) produces the worst outcome among the ten policy rules considered.

Rules that allow for targeting both stage-specific producer inflation rates (Policy Rules 6 and 9) or both PPI and CPI inflation rates (Policy Rules 4 and 5) do substantially better than either the conventional Taylor rule or the exchange rate peg, even if one forgoes the real exchange rate or even the output gap.

The fundamental intuition for these relative welfare rankings is that, with sticky prices, producer price inflation in each stage of production leads to resource misallocation. Thus, a good monetary policy rule should take into account producer price inflation in all stages of production. This intuition appears to be robust to whether we use a balanced trade condition or a risk sharing condition.

Appendix J. Comparative statics: distortions from the stage-specific price stickiness and the elasticity of substitution

If there are different degrees of price stickiness in different stages of production, which one matters more? To shed light on this question, we consider two extreme cases: (i) let the upstream prices be fully flexible (while maintaining the Calvo parameter for the downstream sector at the baseline value), i.e., $\alpha_1 = 0$ and $\alpha_2 = 0.66$; and (ii) let the downstream prices be fully flexible (while keeping the upstream sector Calvo parameter at the baseline value), i.e., $\alpha_1 = 0.66$ and $\alpha_2 = 0$. All other parameters are the same as in Section 4.1. We examine how our results vary with respect to different degrees of openness. We maintain the standard Taylor rule in these exercises.

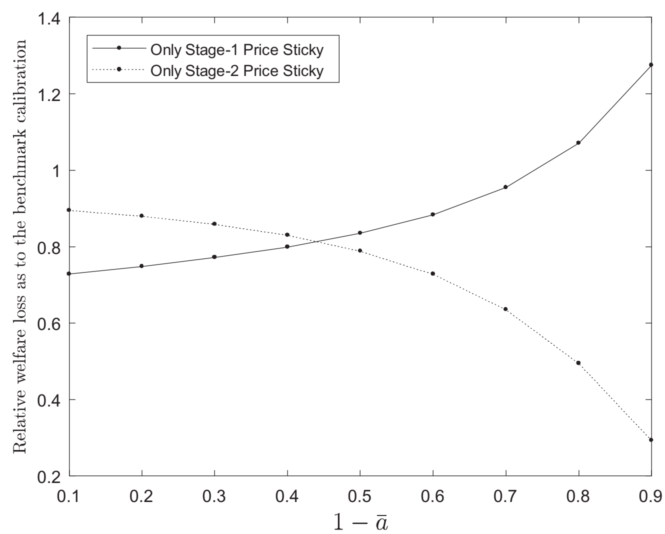

Fig. 5. Relative welfare loss with either upstream-stage price fully flexible or downstream-stage price fully flexible with respect to country openness.

Fig. 5 traces out the welfare loss in the two cases (both relative to the benchmark case, i.e., $\alpha_1 = \alpha_2 = 0.66$).[29] The x-axis represents the degree of openness (or the export share). When the degree of openness is below a threshold, the price stickiness in the downstream stage produces a bigger welfare loss. However, when the economy becomes sufficiently open, the price stickiness in the upstream stage produces more welfare loss.

Since the output of the upstream stage is an input into the downstream stage, the price stickiness of the upstream stage contributes to sluggish output adjustment or resource misallocation in the downstream stage. So, the deviations of the downstream labor allocation and output from the flexible-price equilibrium are greater than those of the upstream stage.

This feature by itself does not imply that the sticky prices in the upstream stage are more important for the overall welfare, because the relative importance of the two stages also depends on their relative employment shares, which in turn depend on the share of intermediate goods in the downstream production. A smaller share of the intermediate goods in the final goods production means a higher share of labor in the downstream stage. For Canada, the intermediate goods share is about 60% (inferred from the World Input-Output Table). Our calibration suggests that when the economy is not very open (including when it is closed), sticky prices in the downstream sector matters more for welfare.

From WIOD data, we calculate that 75% of the countries have an intermediate goods share less than 55%. For these countries, it is also likely the case that sticky prices in the downstream sector produce a greater welfare loss than that sticky prices in the upstream stage, as long as their degree of openness is below some threshold.

As the economy becomes more open, since the upstream sector has to produce for both the world market and the downstream stage at home, the upstream sector employment occupies a progressively larger share in total employment. As a result, the distortion caused by the price stickiness in the upstream stage increases in relative importance. Eventually, when the degree of openness surpasses some threshold, sticky prices in the upstream stage generate a bigger welfare loss.

[29] Notably, when the degree of openness is large, the case of only upstream-stage price being sticky can generate a higher welfare loss than the benchmark calibration with prices being sticky in both stages. The reason is that, the monetary policy reaction function is kept to be the standard Taylor rule in this exercise. When the degree of openness is large enough, the standard Taylor rule is more sub-optimal in the case of only upstream-stage price being sticky.

We also study how the elasticity of substitution affects the welfare at each stage of production, and how it relates to the degree of openness. In general, when the elasticity of substitution is greater, there is more misallocation. We further consider two specific cases: (i) a higher elasticity in the upstream stage, i.e., $\theta_1 = 15$ and $\theta_2 = 10$; and (ii) the opposite case of a higher elasticity in the downstream stage, i.e., $\theta_1 = 10$ and $\theta_2 = 15$. We maintain a classic Taylor rule in both cases.

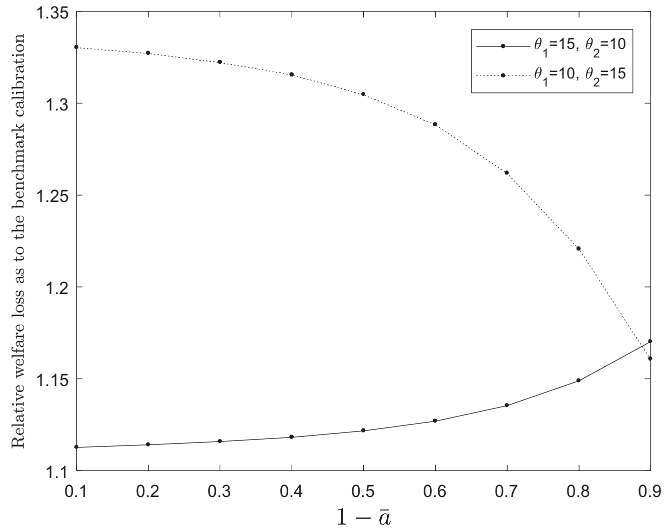

Fig. 6. Relative welfare loss with either higher elasticity of substitution in upstream stage or higher elasticity of substitution in downstream stage with respect to country openness.

Fig. 6 traces out the welfare losses in the two cases (both relative to the benchmark calibration of $\theta_1 = \theta_2 = 10$). The x-axis represents the degree of openness. For reasons similar to the discussion on heterogeneous price stickiness, an increase in the elasticity of substitution in the downstream sector produces a bigger welfare loss than an equivalent increase in the elasticity in the upstream sector as long as the degree of openness is below some threshold. As the economy becomes more open, the labor share of the upstream sector in total employment also increases, and the gap in welfare loss between the two cases narrows. Eventually, when the degree of openness exceeds the threshold, the result flips.

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